Multiscale Fluid Interactions by Composition of Maps (C∘M)

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https://www.imperial.ac.uk/ocean-dynamics-synergy/

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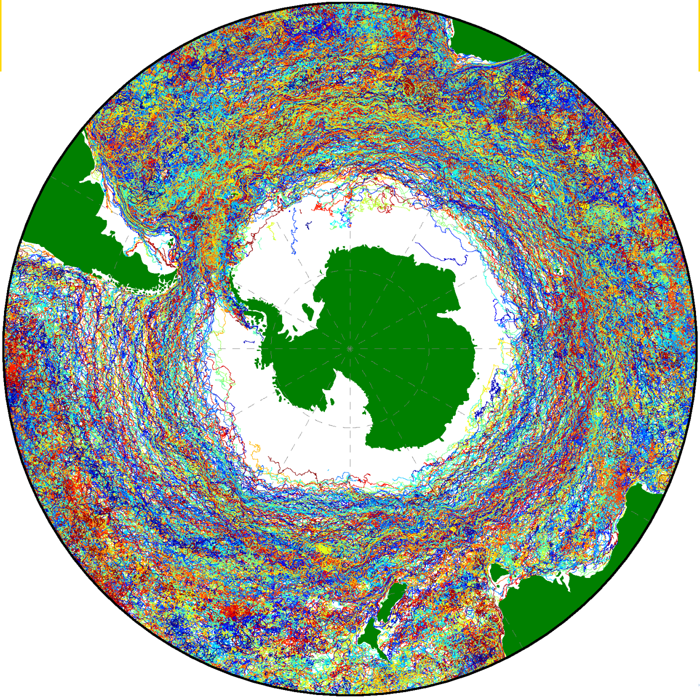

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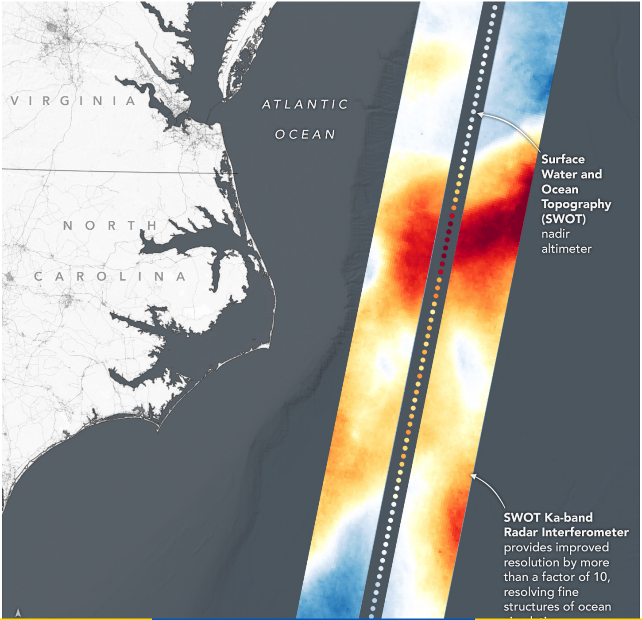
SWOT 1st Light 2023-03-24 Gulf Stream topography



Multi-scale analysis is required for modern satellite data

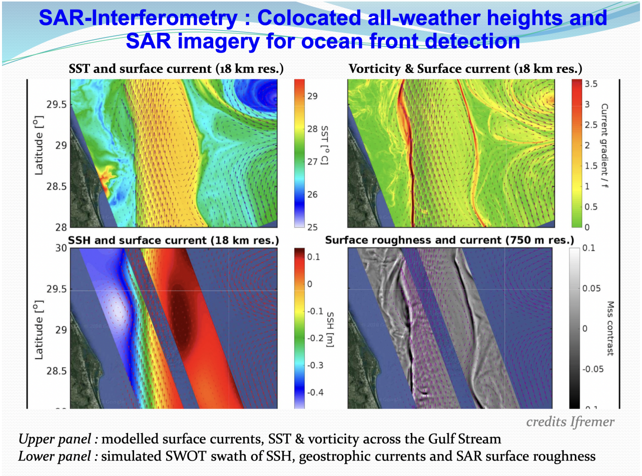


Figure: SWOT will observe many different interacting fluid components.

Oceans have Magnascales, Mesoscales and Sub-mesoscales

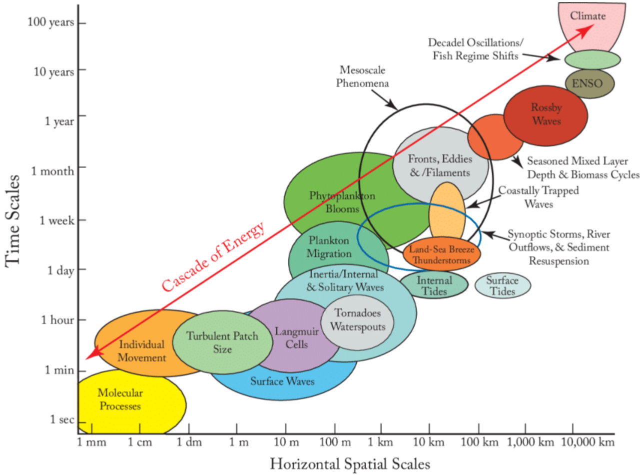


Figure: Oceanic Magna-, Meso- and Submeso scales, cf. Dickey & Bidigare [2005]

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Atmosphere Length/Time scale models (Klein ARFM2010)

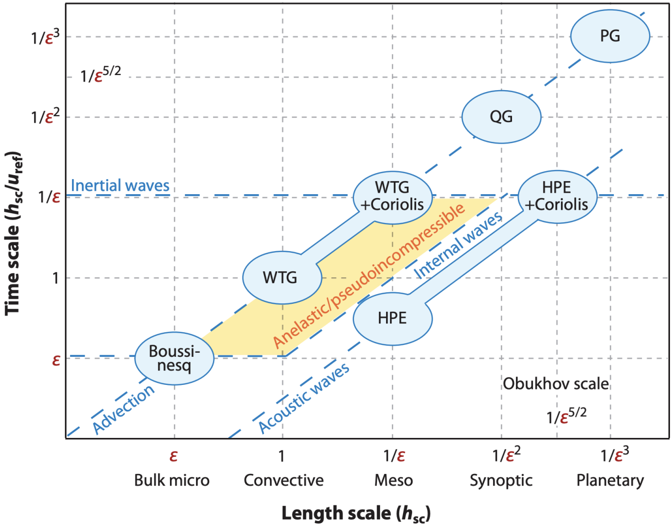

Figure: Magna-, Meso- and Submeso atmospheric phenomena.

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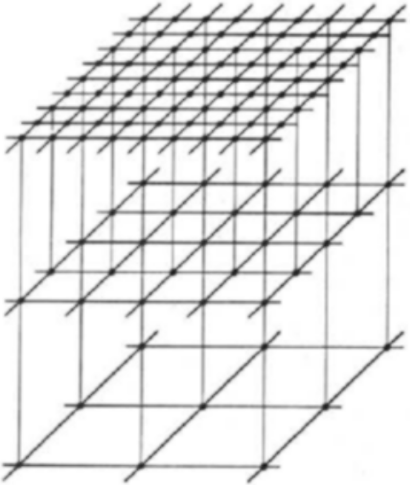
Multiscale analysis involves multiple levels of description



(1) Fine-scale 'Truth' (1024^2)
e.g., from fluid PDE simulations
$\partial_t u + u \cdot \nabla u = -\nabla p, \ \mathrm{div}\, u = 0$.

(2) Coarse-scale avg PDE (64^2)
e.g. space/time average velocity,
$u(x, x, t) := \overline{u(x, x, x, t)}$
This approx introduces uncertainty
of closure problems $\overline{u \cdot \nabla u}(x, x, t)$.

(3) Coarser-scale simulation (4^2)
e.g. Large Eddy Simulation (LES)
LES introduces more uncertainty
due to closure problems.

C∘M has some advantages for multiscale fluid interactions

- **Why?** Satellite data observes effects of multiscale fluid interactions. Regarding multiscale fluid interactions as C∘M is natural, because for smaller scale motions the larger ones are Lagrangian reference frames.
- **How?** Euler-Poincaré variational principles for C∘M provide multicomponent, multiphysics, multiscale, Hamiltonian fluid models. An advantage is that they are very general and coordinate free. A disadvantage is that they require averaging over the smaller scales.
- **What?** Multiscale C∘M models can be applied either by representing expected phenomena, or by projection onto orthonormal modes.

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Why? How? What? and What Next? Details for C∘M

1. **Why C∘M?**
   C∘M fluid dynamics describes nested interactions of multiple DoF.
   The nested physics has a *self-similar* Lie algebraic structure

$$\mathfrak{s} = \mathfrak{g}_1 \circledS \Big(V_1 \oplus \big(\mathfrak{g}_2 \circledS \big(V_2 \oplus (\mathfrak{g}_3 \circledS V_3) \big) \Big).$$

   The nested pattern reveals how to make further extensions of DOF.

2. **How does C∘M work, mathematically?**
   Larger scales **sweep** smaller ones by *push-forward* of C∘M $(g_1 g_2)_*$.

3. **What results arise from the C∘M approach?**
   C∘M variational principle yields **coordinate-free** space-time averaged models that possess a physically sensible Kelvin circulation theorem.

4. **What next for C∘M?**
   Still in progress! Disadvantage: averaging limits applicability.

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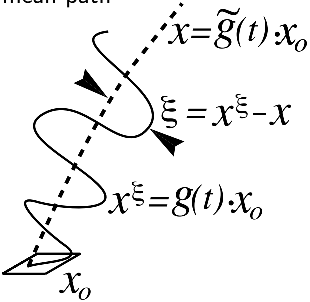
GLM defines fluid velocity at displaced oscillating position

Recall that GLM defines the fluid velocity as $\mathbf{u}^\xi(\mathbf{x},t) := \mathbf{u}(\mathbf{x}+\boldsymbol{\xi}(\mathbf{x},t,t/\epsilon),t)$ at the displaced oscillating position $\mathbf{x}_t + \boldsymbol{\xi}(\mathbf{x}_t, t, t/\epsilon)$ where $\mathbf{x}_t$ is evaluated at the current position $\mathbf{x}$ on a Lagrangian mean path



$$\mathbf{u}^\xi := \frac{D^L \mathbf{x}^\xi}{Dt} := \frac{D^L}{Dt}\Big(\mathbf{x} + \boldsymbol{\xi}(\mathbf{x},t,t/\epsilon)\Big) = \mathbf{u}^L(\mathbf{x},t) + \mathbf{u}^\ell(\mathbf{x},t,t/\epsilon)$$

$$\text{with} \qquad \frac{D^L}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}^L \cdot \frac{\partial}{\partial \mathbf{x}} \quad \text{and} \quad \mathbf{u}^\ell := \frac{D^L \boldsymbol{\xi}}{Dt}\,.$$

GLM then defines the *Lagrangian mean velocity* as $\overline{\mathbf{u}^\xi}(\mathbf{x},t) = \mathbf{u}^L(\mathbf{x},t) = \dot{\widetilde{g}}(t)\widetilde{g}(t)^{-1}\mathbf{x}$, where $\overline{(\,\cdot\,)}$ is a time, or phase average at fixed *Eulerian* coordinate $\mathbf{x}$.

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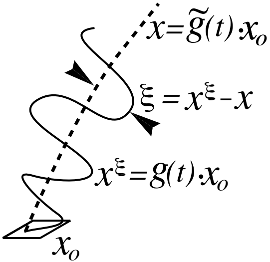
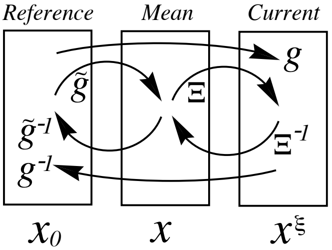
GLM arises from the tangent of composition of two maps

$$\mathbf{X}_t = g_t\mathbf{x}_0 = (Id + \alpha\Xi_t)\circ\bar{g}_t\mathbf{x}_0 =: \mathbf{x}_t + \alpha\boldsymbol{\xi}(\mathbf{x}_t, t)\,,$$





$$U_t(\mathbf{X}_t, t) := \frac{d\mathbf{X}_t}{dt} = \dot{g}_t g_t^{-1}\mathbf{X}_t = \frac{d\mathbf{x}_t}{dt} + \alpha\Big(\partial_t\boldsymbol{\xi}(\mathbf{x}_t, t) + \frac{\partial\boldsymbol{\xi}}{\partial x_t^j}\frac{dx_t^j}{dt}\Big)$$

$$= \dot{\bar{g}}_t\bar{g}_t^{-1}\mathbf{x}_t + \alpha\Big(\partial_t\boldsymbol{\xi}(\mathbf{x}_t, t) + \frac{\partial\boldsymbol{\xi}}{\partial x_t^j}\cdot(\dot{\bar{g}}_t\bar{g}_t^{-1}x_t^j)\Big)$$

$$=: \mathbf{u}_L(\mathbf{x}_t, t) + \alpha\frac{d}{dt}\boldsymbol{\xi}(\mathbf{x}_t, t)$$

Recovers GLM velocity.

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Example: C∘M result for wave-current interaction

In Lie-Poisson bracket form, the wave-current equations studied lately is

$$\frac{\partial}{\partial t}\begin{bmatrix} m \\ D \\ \rho \\ J \\ N \end{bmatrix} = -\begin{bmatrix} \mathrm{ad}^*_{\square}\, m & \square \diamond D & \square \diamond \rho & \square \diamond J & \square \diamond N \\ \mathcal{L}_{\square} D & 0 & 0 & 0 & 0 \\ \mathcal{L}_{\square} \rho & 0 & 0 & 0 & 0 \\ \mathrm{ad}^*_{\square}\, J & 0 & 0 & \mathrm{ad}^*_{\square}\, J & \square \diamond N \\ \mathcal{L}_{\square} N & 0 & 0 & \mathcal{L}_{\square} N & 0 \end{bmatrix} \begin{bmatrix} \delta H/\delta m \\ \delta H/\delta D \\ \delta H/\delta \rho \\ \delta H/\delta J \\ \delta H/\delta N \end{bmatrix},$$

Fluid variables are: momentum $m = D\rho u$, with Eulerian velocity u, scalar mass density ρ and volume form D.

Wave variables are canonically conjugate, (ϕ, N) by $\mathbf{J} := N\nabla\phi$.

The Lie-Poisson bracket for wave-current dynamics is dual to Lie algebra,

$$\mathfrak{s} = \mathfrak{g}_1 \circledS \Big(V_1 \oplus \big(\mathfrak{g}_2 \circledS V_2 \big) \Big) .$$

Hence, the Lie-Poisson bracket $\{F, H\}$ satisfies the Jacobi identity.

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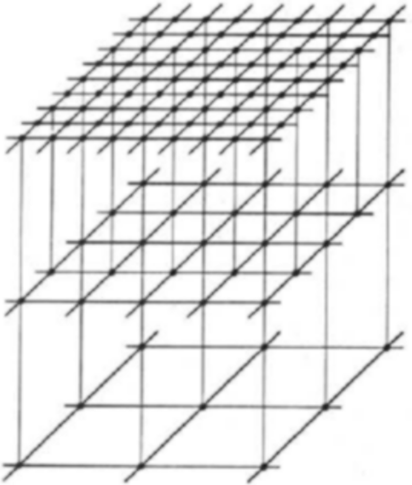
Keep this figure in mind



(1) Fine-scale 'Truth' (1024^2)
e.g., from fluid PDE simulations
$\partial_t u + u \cdot \nabla u = -\nabla p$, $\mathrm{div}\, u = 0$

(2) Coarse-scale avg PDE (64^2)
e.g. space/time average velocity, u
This approx introduces uncertainty.

(3) Coarser-scale simulation (4^2)
e.g. Large Eddy Simulation (LES)
LES introduces more uncertainty.

Now consider composition of *multiple maps* $g_1 \dots g_N$

If the Lagrangian had relabelling symmetry for each successive map, then the action integral in Hamilton's principle would take the form,[1]

$$S = \int_{t_1}^{t_2} L(g_1, \dot{g}_1, a_1^0; g_2, \dot{g}_2, a_2^0; g_3, \dot{g}_3, a_3^0; \dots; g_N, \dot{g}_N, a_N^0)\, dt$$

$$S = \int_{t_1}^{t_2} L(\dot{g}_1 g_1^{-1}, a_1^0 g_1^{-1}; (\dot{g}_2 g_2^{-1}) g_1^{-1}, (a_2^0 g_2^{-1}) g_1^{-1}; \quad (\dot{g}_3 g_3^{-1}) g_2^{-1} g_1^{-1}, (a_3^0 g_3^{-1}) g_2^{-1} g_1^{-1}; \dots \, dt$$

$$S_{red} =: \int_{t_1}^{t_2} \ell(u_1, a_1; u_2 g_1^{-1}, a_2 g_1^{-1}; u_3 g_2^{-1} g_1^{-1}, a_3 g_2^{-1} g_1^{-1}; \dots)\, dt$$

$$=: \int_{t_1}^{t_2} \ell(u_1, a_1; g_{1*} u_2, g_{1*} a_2; (g_1 g_2)_* u_3, (g_1 g_2)_* a_3; \dots)\, dt$$

We restrict to the case that $(u_k := \dot{g}_k g_k^{-1}, a_k := a_1^0 g_k^{-1})$, for $k = 1, 2, 3$. Colour coded as $k = 1$, $k = 2$, $k = 3$.

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[1] Colours denote spatial domains. E.g., $u_3(x_1, x_2, x_3, t)$ and $dV = dx_1^3 dx_2^3 dx_3^3$.

Euler-Poincaré variational relations for C∘M dynamics

Variational relations for nested degrees of freedom exist,
because of a **Lie chain rule (LCR)**. E.g., in varying *advected quantities*,

$$\delta a_k(t) =: a_k'(t) := \partial_\epsilon a_k(t,\epsilon)\Big|_{\epsilon=0} :=: \big(g_{k*}(t)a_k^0\big)'$$

$$\text{By LCR} \quad =: -\mathcal{L}_{g_k' g_k^{-1}(t)} a_k(t) = -\mathcal{L}_{w_k(t)} a_k(t)\,, \qquad w_k := g_k' g_k^{-1}(t)$$

Euler-Poincaré variational relations for velocities $u_k := \dot g_k g_k^{-1} =: g_{k*}\dot g_k$ are obtained from equality of cross derivatives $\dot g_k' = g_k'^{\,\cdot}$ and LCR. Namely,

$$u_1' - (\partial_t - \mathrm{ad}_{u_1})w_1 = 0,$$

$$g_{1*}\big(u_2' - (\partial_t - \mathrm{ad}_{u_1})w_2 + \mathcal{L}_{w_1}u_2\big) = 0,$$

$$(g_1g_2)_*\big(u_3' - (\partial_t - \mathrm{ad}_{u_1+u_2})w_3 + \mathcal{L}_{w_1+w_2}u_3\big) = 0.$$

- The further sequence of EP variational relations follows a clear pattern.
- Larger scales **sweep** smaller ones by C∘M push-forward $(g_1g_2)_*$.

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Hamilton's variational principle $\delta S_{red} = 0$ for multiple C∘M

Hamilton's principle for C∘M velocity variations u_1', u_2', and u_3' yields

$$
\begin{aligned}
0 = \delta S_{red} = \int_{t_1}^{t_2} & \left\langle \frac{\delta \ell}{\delta u_1} \,,\, (\partial_t - \mathrm{ad}_{u_1}) w_1 \right\rangle_{L^2} \\
& + \left\langle \frac{\delta \ell}{\delta u_2} \,,\, (\partial_t - \mathrm{ad}_{u_1}) w_2 + \mathcal{L}_{w_1} u_2 \right\rangle_{L^2} \\
& + \left\langle \frac{\delta \ell}{\delta u_3} \,,\, (\partial_t - \mathrm{ad}_{u_1+u_2}) w_3 + \mathcal{L}_{w_1+w_2} u_3 \right\rangle_{L^2} \\
& + \left\langle \frac{\delta \ell}{\delta a_1} \,,\, -\mathcal{L}_{w_1} a_1 \right\rangle_{L^2} + \left\langle \frac{\delta \ell}{\delta a_2} \,,\, -\mathcal{L}_{w_1+w_2} a_2 \right\rangle_{L^2} \\
& + \left\langle \frac{\delta \ell}{\delta a_3} \,,\, -\mathcal{L}_{w_1+w_2+w_3} a_3 \right\rangle_{L^2} dt
\end{aligned}
$$

Three EP equations follow by *collecting coefficients* of w_1, w_2, and w_3 then setting each coefficient equal to zero.

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Three (colour-coded) Euler-Poincaré motion equations

EP equations for $k = 1, 2, 3$ emerge after *collecting coefficients* of w_1, w_2, w_3 with **diamond operation** $(\diamond)$ defined by $\langle b \diamond a \,,\, w \rangle_{\mathfrak{X}} := \langle b \,,\, -\mathcal{L}_w a \rangle_V$. Averaging over successive scales [Holm-Tronci 2012] leads to

$$
\begin{aligned}
0 = \delta S_{red} = &\\
- \int_{t_1}^{t_2} & \left\langle (\partial_t + \mathrm{ad}^*_{u_1}) \frac{\delta \ell}{\delta u_1} - \frac{\delta \ell}{\delta a_1} \diamond a_1 \,,\, w_1 \right\rangle_{L^2} \\
& - \left\langle \overline{\frac{\delta \ell}{\delta u_2} \diamond u_2} + \overline{\frac{\delta \ell}{\delta a_2} \diamond a_2} + \overline{\overline{\frac{\delta \ell}{\delta u_3} \diamond u_3}} + \overline{\overline{\frac{\delta \ell}{\delta a_3} \diamond a_3}} \,,\, w_1 \right\rangle_{L^2} \\
& + \left\langle (\partial_t + \mathrm{ad}^*_{u_1+u_2}) \frac{\delta \ell}{\delta u_2} - \frac{\delta \ell}{\delta a_2} \diamond a_2 - \overline{\frac{\delta \ell}{\delta u_3} \diamond u_3} - \overline{\frac{\delta \ell}{\delta a_3} \diamond a_3} \,,\, w_2 \right\rangle_{L^2} \\
& + \left\langle (\partial_t + \mathrm{ad}^*_{u_1+u_2+u_3}) \frac{\delta \ell}{\delta u_3} - \frac{\delta \ell}{\delta a_3} \diamond a_3 \,,\, w_3 \right\rangle_{L^2} dt
\end{aligned}
$$

Auxiliary relations hold for the multiscale advected quantities:

$$
a_1 := g_{1*} a_1^0, \qquad a_2 := (g_1 g_2)_* a_2^0, \qquad a_3 := (g_1 g_2 g_3)_* a_3^0
$$

$$
\partial_t a_1 = -\mathcal{L}_{u_1} a_1, \quad \partial_t a_2 = -\mathcal{L}_{(u_1+u_2)} a_2, \quad \partial_t a_3 = -\mathcal{L}_{(u_1+u_2+u_3)} a_3.
$$

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Legendre transform $\Longrightarrow$ *nested* Hamiltonian formulation

The Euler-Poincaré equations may be displayed in Lie-Poisson matrix form with reduced Hamiltonian $h(m_k, a_k) : \Pi_k(\mathfrak{X}^*_k \times V^*_k) \to \mathbb{R}$ with $m_k := \frac{\delta \ell}{\delta u_k}$.

$$
\partial_t \begin{pmatrix} m_1 \\ a_1 \\ m_2 \\ a_2 \\ m_3 \\ a_3 \end{pmatrix} = - \begin{pmatrix}
\mathrm{ad}^*_\square \, m_1 & \square \diamond a_1 & \overline{\square \diamond m_2} & \overline{\square \diamond a_2} & \overline{\overline{\square \diamond m_3}} & \overline{\overline{\square \diamond a_3}} \\
\mathcal{L}_\square a_1 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{L}_\square m_2 & 0 & \mathrm{ad}^*_\square \, m_2 & \square \diamond a_2 & \overline{\square \diamond m_3} & \overline{\square \diamond a_3} \\
\mathcal{L}_\square a_2 & 0 & \mathcal{L}_\square a_2 & 0 & 0 & 0 \\
\mathcal{L}_\square m_3 & 0 & \mathcal{L}_\square m_3 & 0 & \mathrm{ad}^*_\square \, m_3 & \square \diamond a_3 \\
\mathcal{L}_\square a_3 & 0 & \mathcal{L}_\square a_3 & 0 & \mathcal{L}_\square a_3 & 0
\end{pmatrix}
\begin{pmatrix}
\frac{\delta h}{\delta m_1} = u_1 \\
\frac{\delta h}{\delta a_1} = -\frac{\delta \ell}{\delta a_1} \\
\frac{\delta h}{\delta m_2} = u_2 \\
\frac{\delta h}{\delta a_2} = -\frac{\delta \ell}{\delta a_2} \\
\frac{\delta h}{\delta m_3} = u_3 \\
\frac{\delta h}{\delta a_3} = -\frac{\delta \ell}{\delta a_3}
\end{pmatrix} .
$$

The matrix operator defines a Lie-Poisson bracket $\{f, h\} = \langle \mu, [df, dh] \rangle$ on the dual of the following *nested* semidirect product Lie algebra

$$
\mathfrak{s} = \mathfrak{g}_1 \circledS \Big(V_1 \oplus \big(\mathfrak{g}_2 \circledS \big(V_2 \oplus (\mathfrak{g}_3 \circledS V_3) \big) \big) \Big) .
$$

The pattern for further extension to additional DoF is clear.
It shows a sense of Lie algebraic self similarity.

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What's next? Let's discuss!

1. Back reaction (scatter) via feedback in Kelvin circulation theorems
2. Littlewood-Paley Fourier-shell decompositions of interacting fluid scales. Cf. Holm-Tronci [2012]
3. Geometric understanding of multiscale fluid interaction dynamics: Each successively smaller scale regards the previous larger scale as a Lagrange coordinate.
4. The theoretical results for back-reaction of small scales on large scales may guide stochastic models of effects of small scales on large scales.
5. A geometric stochastic approach for multiscale fluid modelling may be introduced via a multiscale Kelvin's circulation theorem.

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Back reaction (scattering) in Kelvin circulation theorems

Deterministic back reaction (scattering) would require modelling the averages of 6 sub-grid scale fluctuating quantities in 2 Kelvin theorems.

$$
\begin{aligned}
\frac{d}{dt}\oint_{c(u_1)} D_1^{-1}\frac{\delta\ell}{\delta u_1} = \oint_{c(u_1)} D_1^{-1}\Bigg(\frac{\delta\ell}{\delta a_1}\diamond a_1 + \overline{\frac{\delta\ell}{\delta u_2}\diamond u_2} + \overline{\frac{\delta\ell}{\delta a_2}\diamond a_2} \\
+ \overline{\overline{\frac{\delta\ell}{\delta u_3}\diamond u_3}} + \overline{\overline{\frac{\delta\ell}{\delta a_3}\diamond a_3}}\Bigg)\,,
\end{aligned}
$$

$$
\frac{d}{dt}\oint_{c(u_1+u_2)} D_2^{-1}\frac{\delta\ell}{\delta u_2} = \oint_{c(u_1+u_2)} D_2^{-1}\left(\frac{\delta\ell}{\delta a_2}\diamond a_2 + \overline{\frac{\delta\ell}{\delta u_3}\diamond u_3} + \overline{\frac{\delta\ell}{\delta a_3}\diamond a_3}\right),
$$

$$
\frac{d}{dt}\oint_{c(u_1+u_2+u_3)} D_3^{-1}\frac{\delta\ell}{\delta u_3} = \oint_{c(u_1+u_2+u_3)} D_3^{-1}\frac{\delta\ell}{\delta a_3}\diamond a_3\,.
$$

$$
(\partial_t + \mathcal{L}_{u_1})D_1 = 0\,, \quad (\partial_t + \mathcal{L}_{u_1+u_2})D_2 = 0\,, \quad (\partial_t + \mathcal{L}_{u_1+u_2+u_3})D_3 = 0\,.
$$

Littlewood-Paley Fourier-shell averages preserve the Kelvin theorem.

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Littlewood-Paley Fourier avg preserves Kelvin's theorem

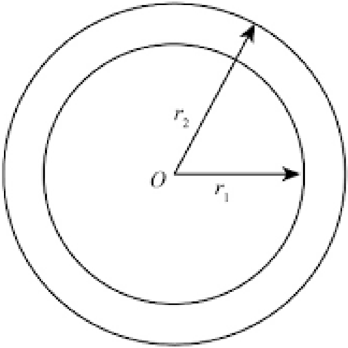

Figure: $k = 1$, $k = 2$, and $k = 3$ scales in Fourier space.

Holm-Tronci [2012] provide details of LP fluid averaging.

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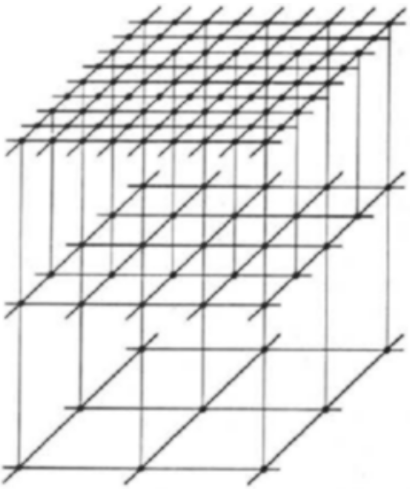
The SALT procedure also involves 3 levels of description



(1) Fine-scale 'Truth' (1024^2)
e.g., from fluid PDE simulations
$\partial_t u + u \cdot \nabla u = -\nabla p, \ \mathrm{div}\, u = 0$

(2) Coarse-scale SALT SPDE (64^2)
for stochastic 2D Euler velocity, u
$\mathbf{d}u + \mathbf{d}x_t \cdot \nabla u + u_j \nabla \mathbf{d}x_t{}^j = -\nabla p \, dt$
$\mathbf{d}x_t = u_t(x_t)dt + \sum \xi_i(x_t) \circ dW_t^i$
Stochastic advection velocity, $\mathbf{d}x_t$
This step quantifies uncertainty.

(3) Particle filtering of data (4^2)
$\mathbf{d}x_t = u_t(x_t)dt + \sum \xi_i(x_t) \circ dW_t^i$
$\mathbf{d}y_t = a(x_t)dt + b(x_t) \circ dB_t$
This step reduces uncertainty.

Pollock

Ocean

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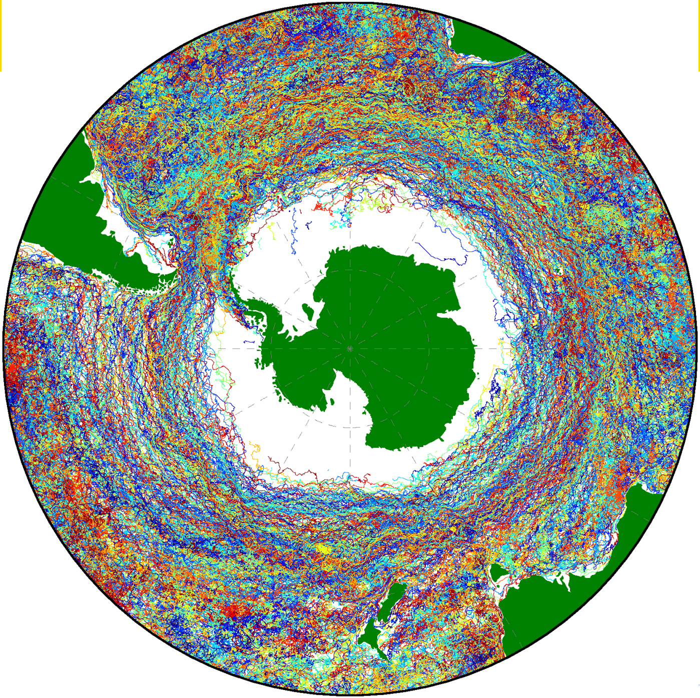

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Alternative: Stochastic geometric mechanics approach

One could model effects of smaller scales on larger scales by introducing SALT noise in Kelvin's circulation laws, specifically by taking the small-scale effects of $k=2$ and $k=3$ as stochastic processes,

$$\mathrm{d}x_k := u_k dt + \xi_k(x)\circ dW_t \quad \text{for} \quad k=2, k=3\,.$$

Stochastic Advection by Lie Transport (SALT) in Kelvin's theorem is [2]

$$\mathrm{d}\oint_{c(\mathrm{d}x_1)} D_1^{-1}\frac{\delta\ell}{\delta u_1} = \oint_{c(\mathrm{d}x_1)} D_1^{-1}\left(\frac{\delta\ell}{\delta a_1}\diamond a_1 + \zeta_2(x)\circ dW_t + \zeta_3(x)\circ dW_t\right),$$

$$\mathrm{d}\oint_{c(\mathrm{d}x_1+\mathrm{d}x_2)} D_2^{-1}\frac{\delta\ell}{\delta u_2} = \oint_{c(\mathrm{d}x_1+\mathrm{d}x_2)} D_2^{-1}\left(\frac{\delta\ell}{\delta a_2}\diamond a_2 + \zeta_3(x)\circ dW_t\right)\,.$$

and the volume elements D_1, D_2, D_3 are stochastically advected, as

$$(\mathrm{d}+\mathcal{L}_{\mathrm{d}x_1})D_1 = 0\,, \quad (\mathrm{d}+\mathcal{L}_{\mathrm{d}x_1+\mathrm{d}x_2})D_2 = 0\,.$$

[2] Fluctuations $\frac{\delta\ell}{\delta u_2}\diamond u_2 = \mathrm{ad}^*_{u_2}\frac{\delta\ell}{\delta u_2}$ & $\frac{\delta\ell}{\delta u_3}\diamond u_3 = \mathrm{ad}^*_{u_3}\frac{\delta\ell}{\delta u_3}$ are modelled as stochastic forces.

What have we been discussing in this talk today?

(i) Began by formulating GLM as the composition by push-forward of two smooth invertible maps in Hamilton's principle for Eulerian fluids.

(ii) Extended C∘M so that internal dynamics of several successive fluid components are transported by the combined actions of those that came before and they combine to transport all those that come after.

(iii) The advantage of the considerations here is their generality. C∘M yields coordinate-free representations of multicomponent, multiphysics non-dissipative fluid dynamics common in oceanic flows.

(iv) The preservation of Kelvin's theorem in the C∘M geometric averaging and closure approach also provides a basis for stochastic modelling. Namely, it identifies what physical terms are modelled by stochasticity.

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What's next? Let's discuss!

https://www.imperial.ac.uk/ocean-dynamics-synergy/

Thanks for listening!

More papers along these lines with up-to-date references are at ORCID:

https://orcid.org/0000-0001-6362-9912

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