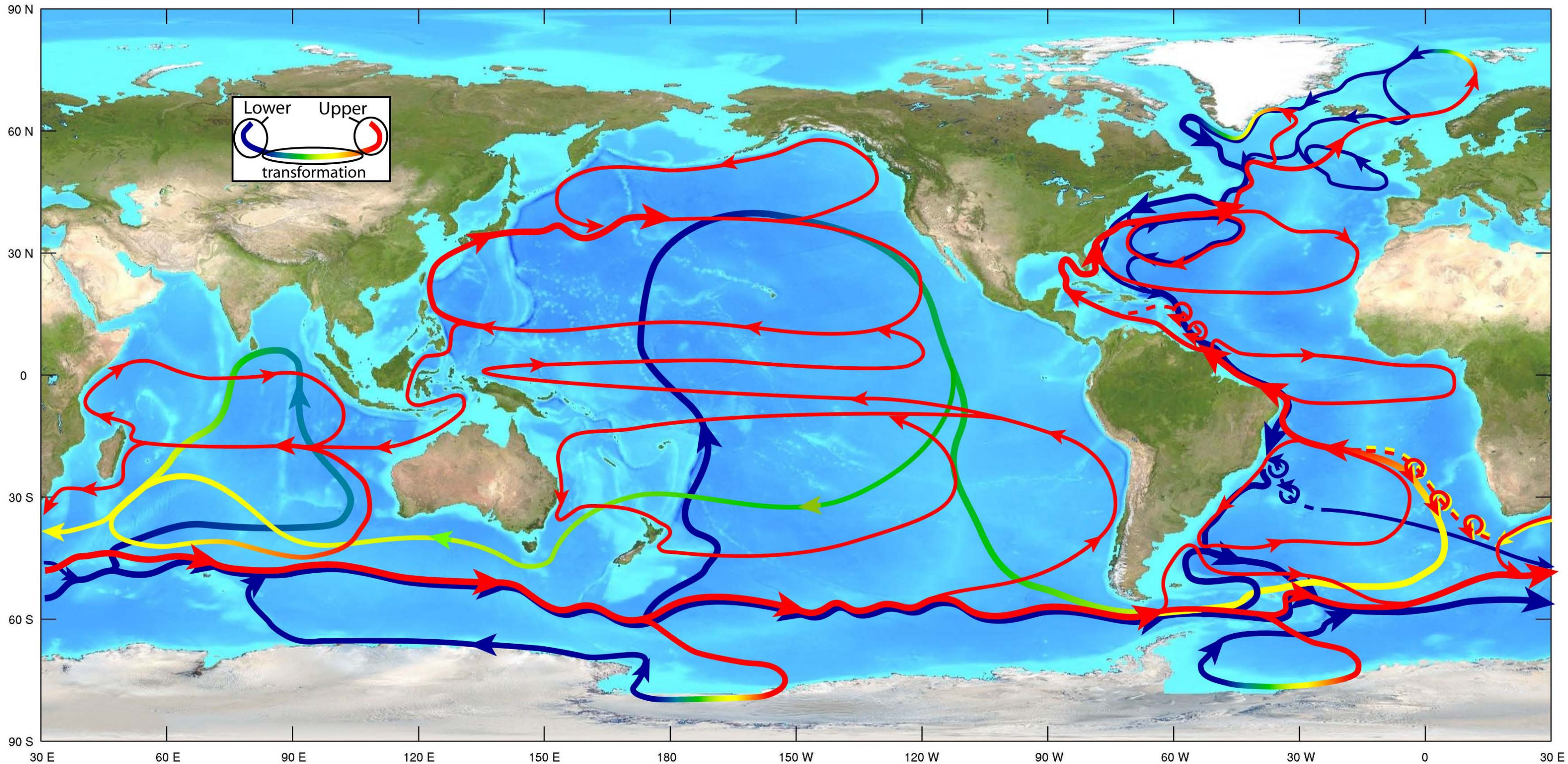
Parameterized internal wave mixing in three ocean general circulation models

Nils Brüggemann[1], Martin Losch[2], Patrick Scholz[2], Friederike Pollmann[4], Sergey Danilov[2], Oliver Gutjahr[1], Johann Jungclaus[1], Nikolay Koldunov[2], Peter Korn[1], Dirk Olbers[2,3], and Carsten Eden[1]

[1] MPI, Hamburg, [2] AWI, Bremerhaven,
[3]Universität Bremen, [4] Universität Hamburg

ocean circulation

- horizontal wind-driven circulation and meridional overturning circulation (MOC)
- surface currents in red, deep circulation in blue



from Lumpkin (2012)

Sandström's inference

- Bjerknes' circulation theorem for circulation $C = \oint_\Gamma (\boldsymbol{u} + \boldsymbol{\Omega} \times \boldsymbol{r}) \cdot d\boldsymbol{s}$

$$\frac{D\boldsymbol{u}}{Dt} = -2\boldsymbol{\Omega} \times \boldsymbol{u} - \frac{1}{\rho}\boldsymbol{\nabla} p - \boldsymbol{\nabla}\Phi + \boldsymbol{F} \;\rightarrow\; \frac{DC}{Dt} = \oint_\Gamma d\boldsymbol{s} \cdot \left(-\frac{1}{\rho}\boldsymbol{\nabla} p + \boldsymbol{F} \right)$$

where $\boldsymbol{\Omega}$ denotes Earth's rotation and Γ a closed curve

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- no driving for heating and cooling at same depth $\rightarrow$ Sandström (1908)

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- atmosphere is like a heat engine, but ocean is like a refrigerator
- ocean's MOC driven by direct mechanical work (surface wind stress)
  or small-scale mixing in interior by breaking internal gravity waves

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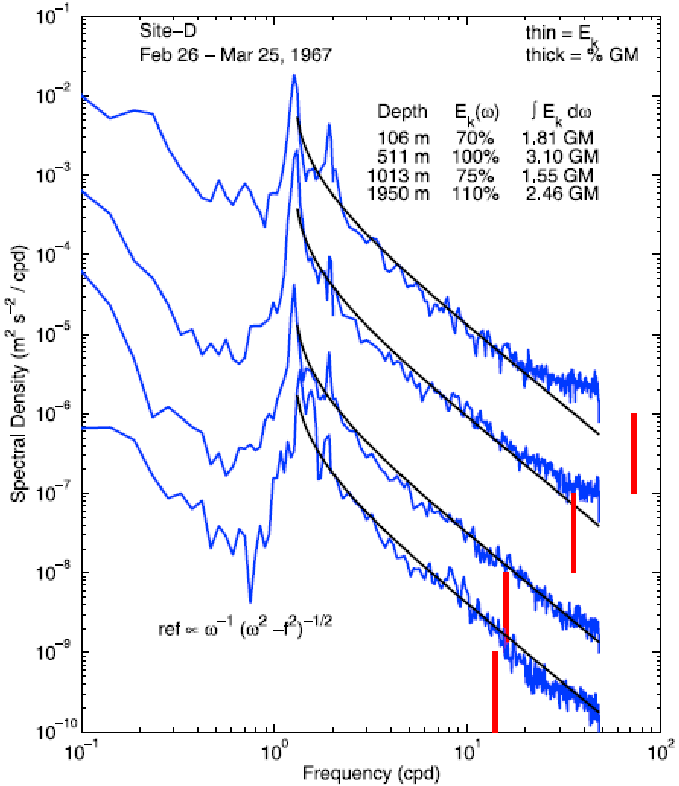
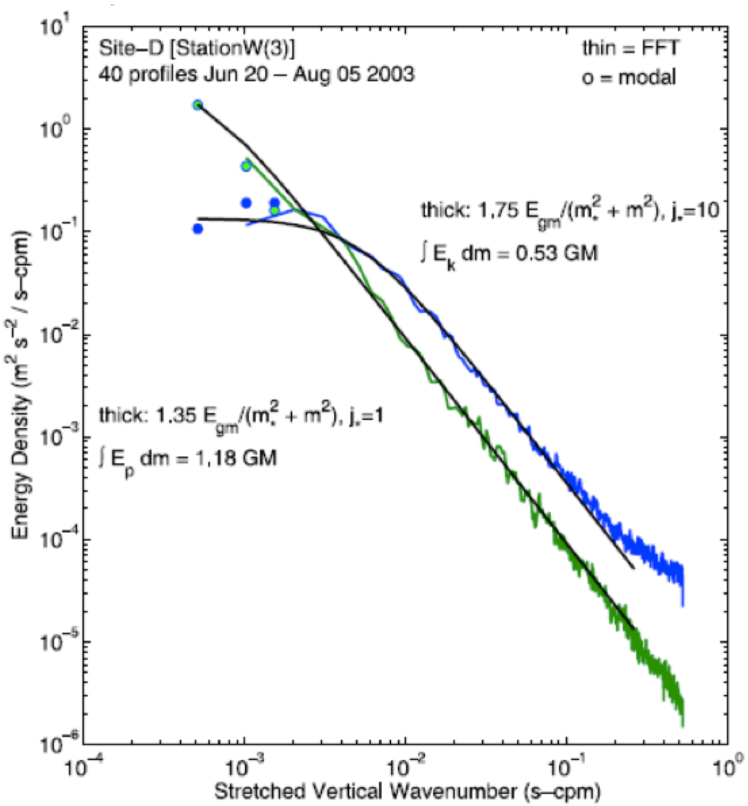
continuous gravity wave spectra

- spectral slopes in frequency and wave number close to -2
  $\rightarrow$ so-called Garrett-Munk (GM) spectrum for internal waves





from Polzin and Lvov (2011)

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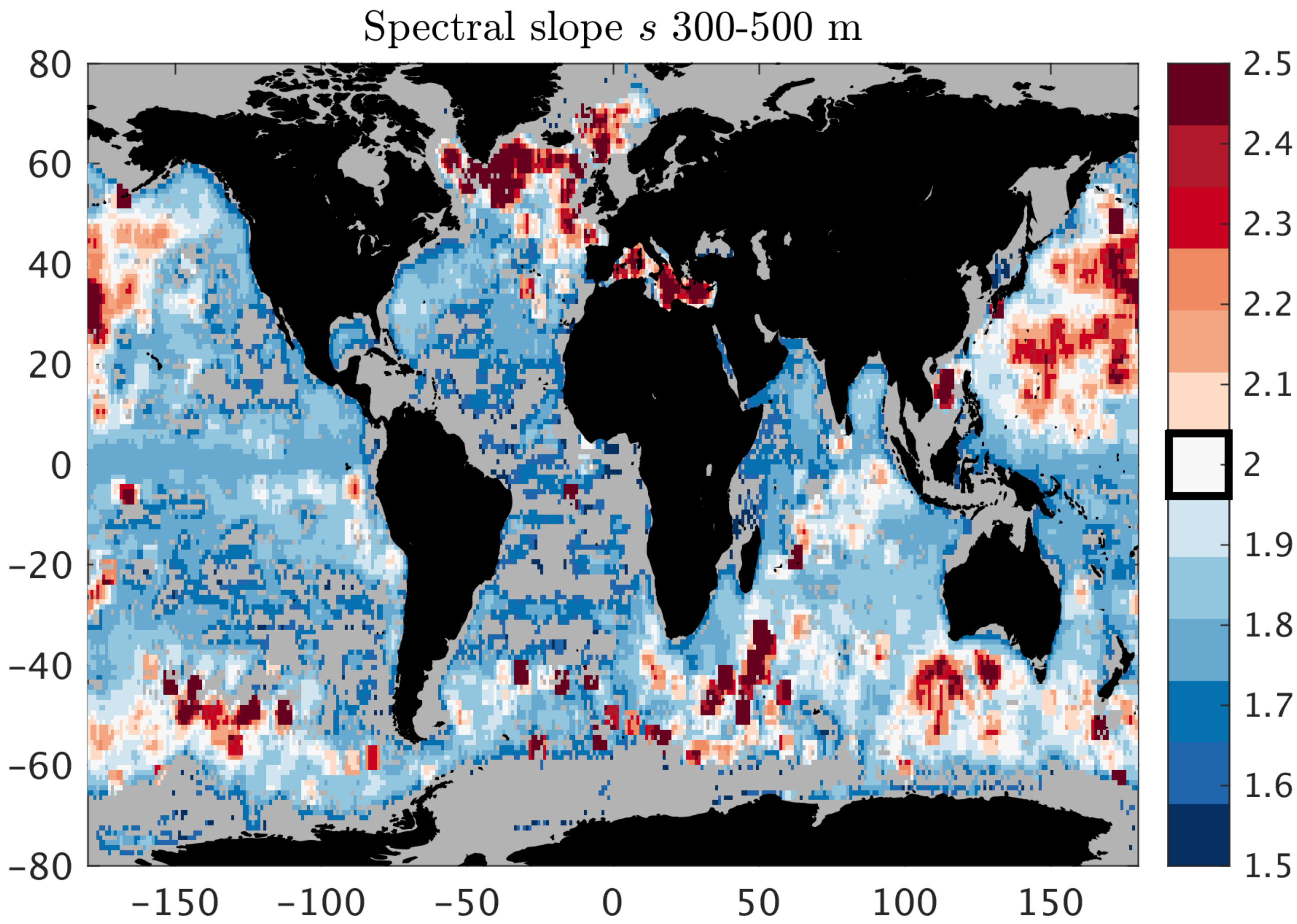

- global slope distribution of vertical wavenumber from ARGO floats
- spectral slopes in vertical wavenumber are close to -2

from Pollmann (2020)

radiative transfer equation for gravity waves

- gravity waves propagating through slowly changing environment

$$\omega = \Omega(\boldsymbol{x}, z, \boldsymbol{k}, m)\ ,\ \ \dot{\boldsymbol{x}} = \boldsymbol{\nabla}_{\boldsymbol{k}}\Omega\ ,\ \ \dot{z} = \partial_m \Omega\ ,\ \ \dot{\boldsymbol{k}} = -\boldsymbol{\nabla}_{\boldsymbol{x}}\Omega\ ,\ \ \dot{m} = -\partial_z \Omega$$

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- co-integrate radiative transfer equation in ocean model
  $\rightarrow$ predict how waves behave, but six-dimensions are too many
  $\rightarrow$ reduce complexity by integration in wavenumber space

IDEMIX concept

- reduce complexity by integration in wavenumber space

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- assume $\mathcal{E} \approx EA$ with total wave energy $E(\boldsymbol{x}, z, t) = \int \mathcal{E}\, d\boldsymbol{k} dm$ and $\int A\ d\boldsymbol{k} dm = 1$ with spectral shape $A(\boldsymbol{k}, m)$ as in GM spectrum

$$\int \dot{z}\, \mathcal{E}\, d\boldsymbol{k} dm = c\, E \quad , \quad c = \int \dot{z}\, \mathcal{E}\, d\boldsymbol{k} dm / E \approx \int \dot{z}\, A\, d\boldsymbol{k} dm$$

with bulk vertical group velocity c, and similar for other terms

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- to treat S turns out to better define energies of up- and downward propagating waves

$$\begin{aligned} E^{\pm} &= \int \int_{-\infty}^{\infty} \max(\pm\sigma, 0) \mathcal{E}\, d\boldsymbol{k} dm \\ c^{\pm} &= \int \int_{-\infty}^{\infty} \max(\pm\sigma, 0) \dot{z}\, \mathcal{E}\, d\boldsymbol{k} dm / E^{\pm} \quad (= c) \end{aligned}$$

with $\sigma = \text{sign}(\dot{z})$

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- several different version of IDEMIX focus on different aspects, here treatment of S (IDEMIX; Internal wave Dissipation, Energy and MIXing)

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- ignore horizontal propagation and wave-mean flow interaction (Olbers and Eden, 2013)

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- rewrite for $E = E^+ + E^-$ and $\Delta E = E^+ - E^-$

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closure for $\int S d\boldsymbol{k} dm$:
- dissipation in S acts to take away total energy with law μE^2
- it also damps asymmetries in up/downward propagating wave energy at time scale τ

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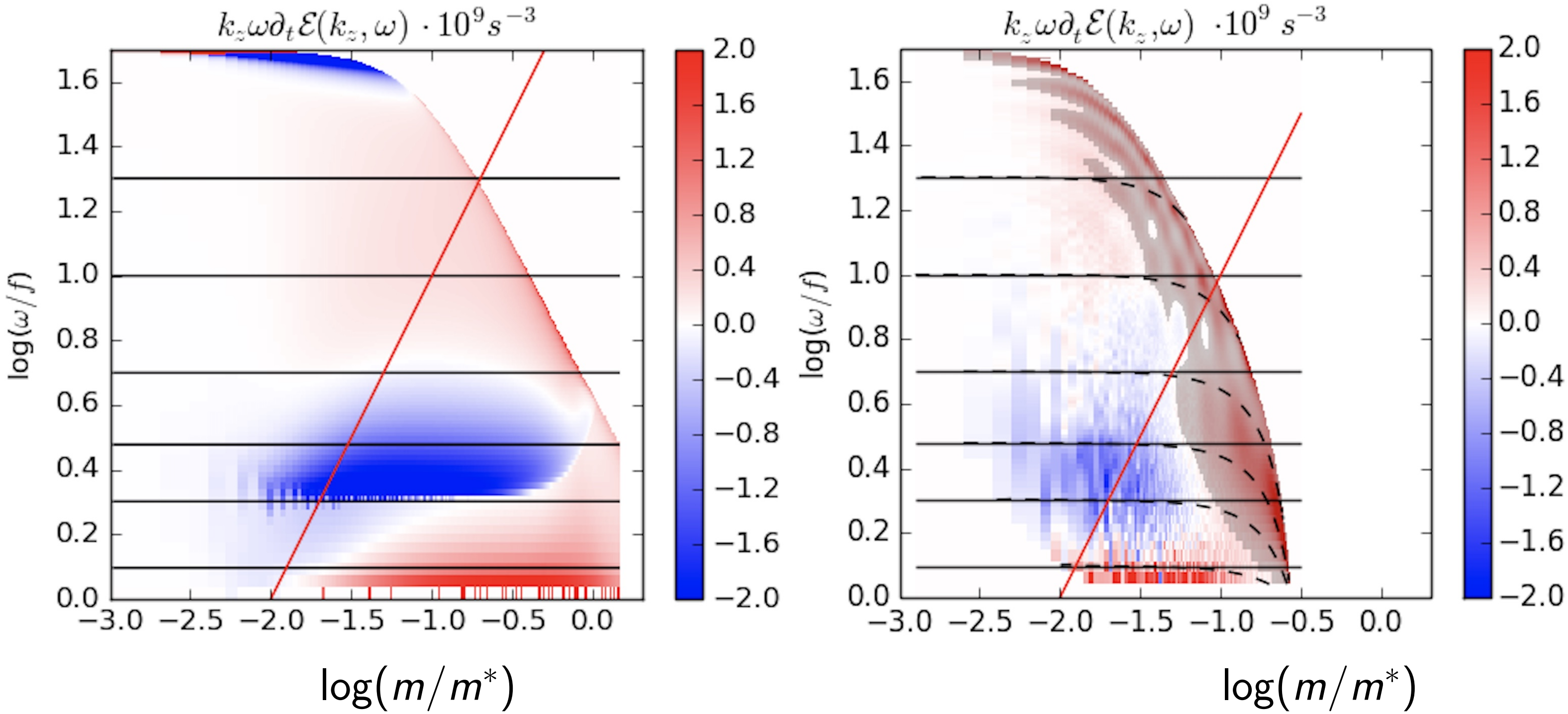
- boundary conditions for vertical energy flux $c\tau\partial_z(c\, E)$ at surface and bottom by oscillatory surface Ekman pumping and tidal flow over bottom

energy transport by wave-wave interaction



Eden, Olbers, Pollmann (2020)

- estimate of $\partial_t \mathcal{E}$ by wave-wave interaction (part of S) in GM spectrum
- left: with scattering integral/Hasselmann's weak interaction theory
  right: with numerical model

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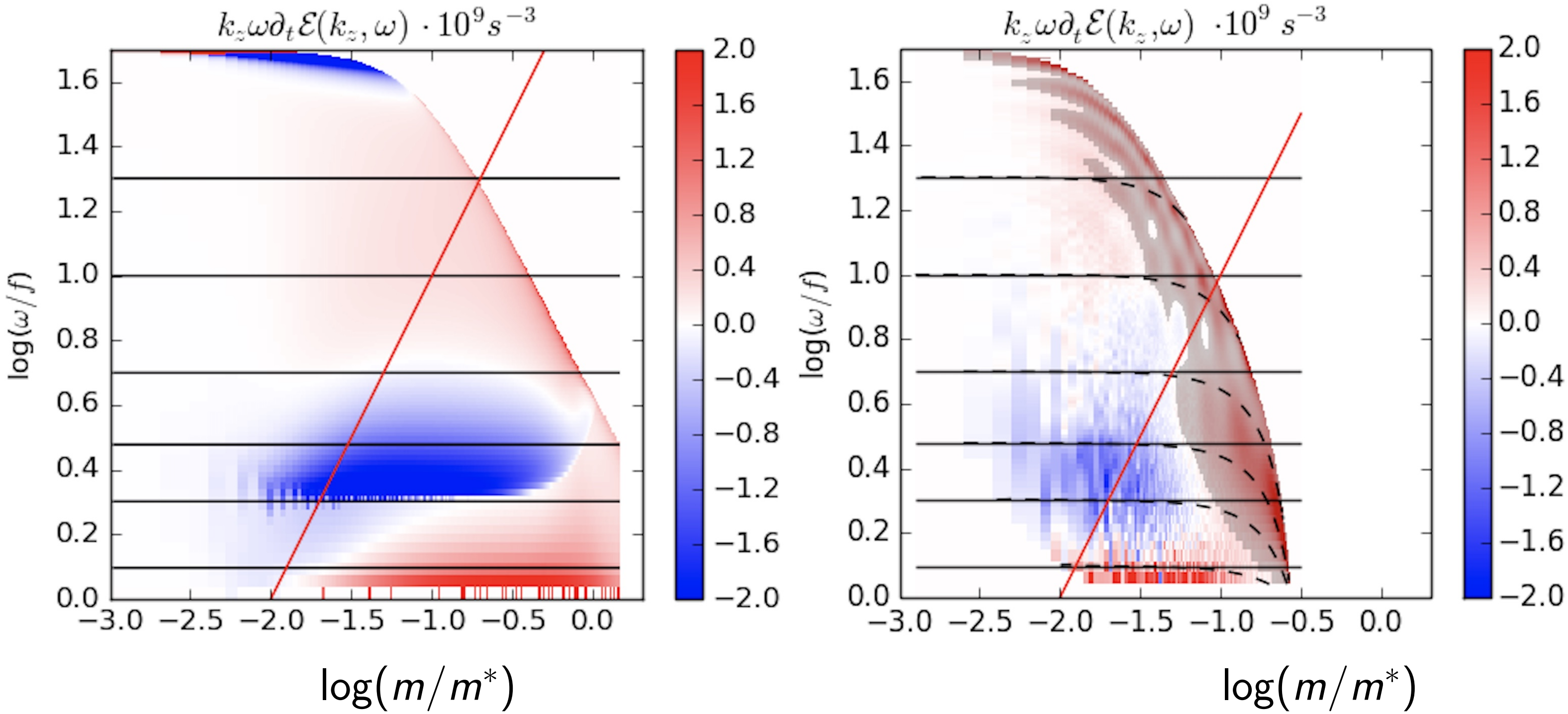

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- estimate of $\partial_t\mathcal{E}$ by wave-wave interaction (part of S) in GM spectrum
- left: with scattering integral/Hasselmann's weak interaction theory
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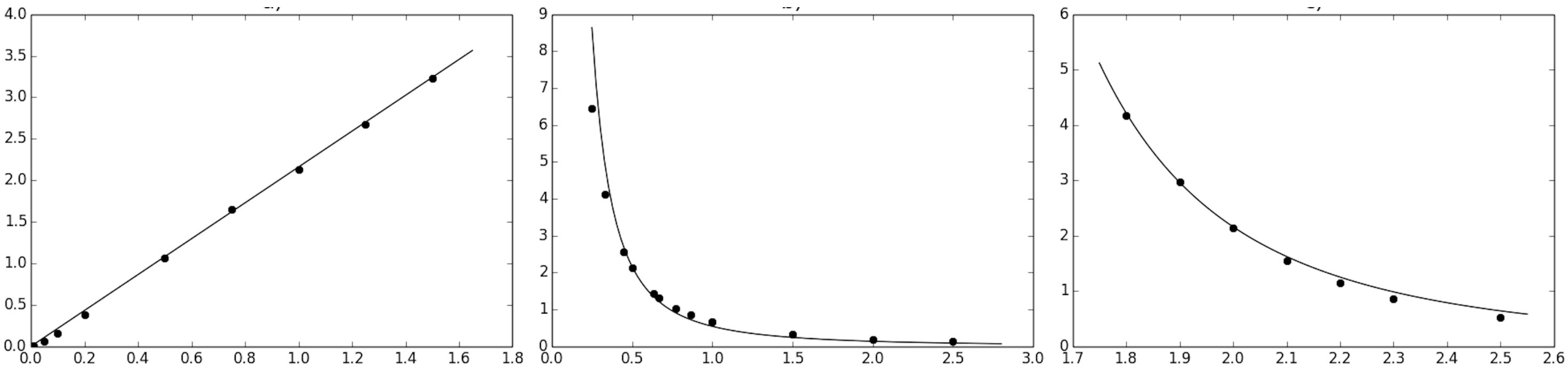
- energy loss at $2f < \omega < 3f$ by PSI,
  energy gain at lower ω but larger $m \rightarrow$ wave breaking/dissipation

parameterisation for wave-wave interactions/dissipation



Coriolis parameter $f/[10^4\,\mathrm{s}^{-1}]$ bandwidth $c_*/[\mathrm{m/s}]$ and slope s of GM spectrum

from Eden, Pollmann, and Olbers (2019)

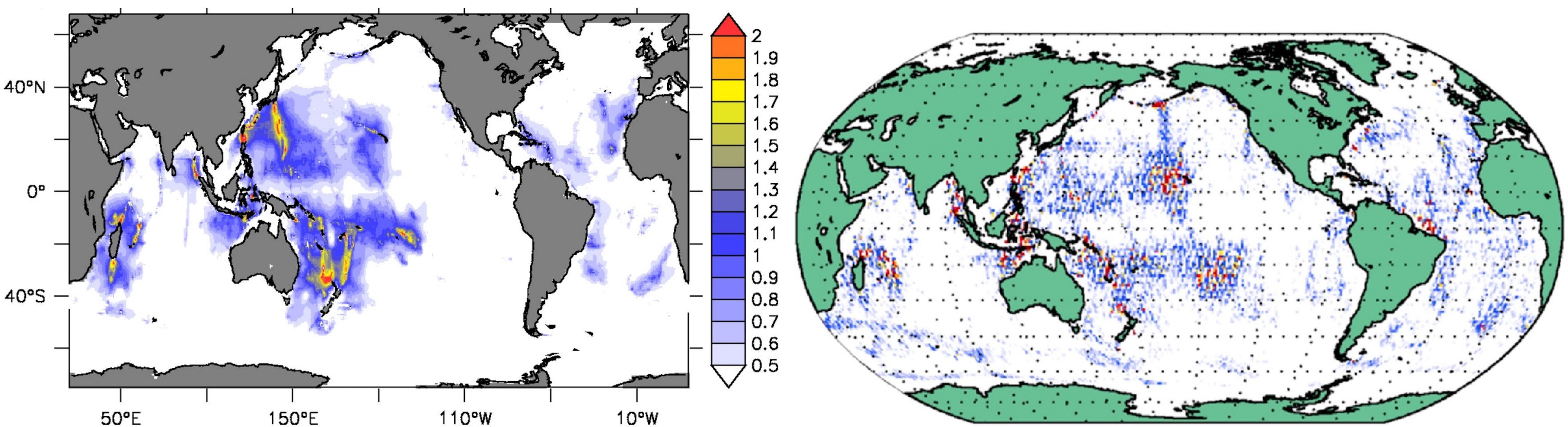
- $\int d\omega dm\, |\partial_t \mathcal{E}^{\pm}|_{2<\omega/f<3}$ (dots) vs. **wave dissipation parameterisation** μE^2 (lines)

$$\mu E^2 = 0.6\, f\, c_\star^{-2}\, (s-1)^{-3} \left(\int d\omega dm \mathcal{E}^{\pm} \right)^2$$

- excellent comparison of dissipation parameterisation

zoo of IDEMIX models

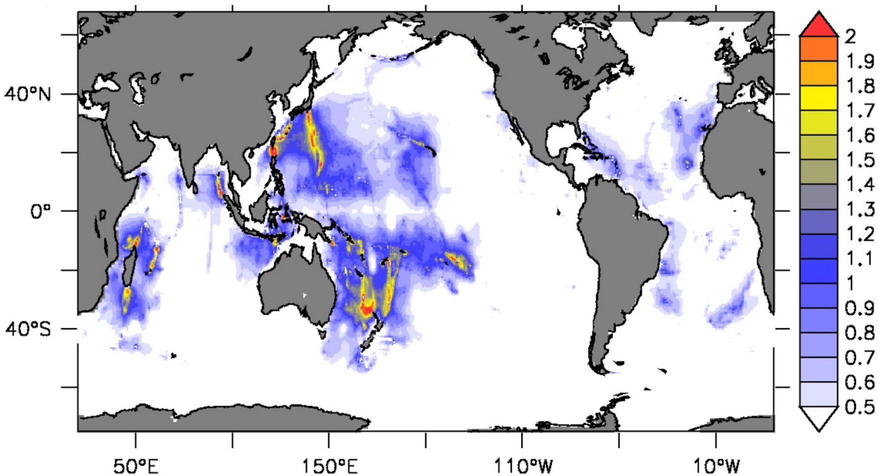
- Olbers and Eden (2013): simple IDEMIX as vertical mixing parameterisation
- Eden and Olbers (2014): extended IDEMIX with resolved horizontal propagation

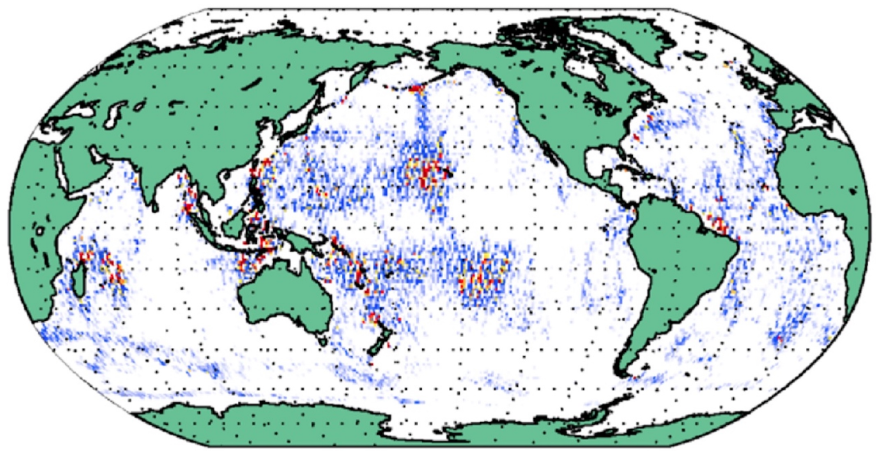


- rms sea surface height due to internal tides in IDEMIX (2014) (left), observations (right)

zoo of IDEMIX models

- Olbers and Eden (2013): simple IDEMIX as vertical mixing parameterisation
- Eden and Olbers (2014): extended IDEMIX with resolved horizontal propagation
- Eden and Olbers (2017): IDEMIX for Rossby waves (meso-scale eddies)
- Olbers and Eden/Eden and Olbers (2017): with wave-mean flow interaction
- Quinn, Eden, Olbers (2020): same but for atmosphere and with critical layers
- Eden, Olbers, Eriksen (2021): IDEMIX for lee waves
- Olbers et al (2023): IDEMIX with energy and bandwidth equation





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- co-integrate also equation for small-scale turbulent kinetic E_{tke} energy (Gaspar et al 1990)

$$\partial_t E_{tke} = \partial_z c_{tke} K_m \partial_z E_{tke} + K_m\left(\partial_z \boldsymbol{u}\right)^2 + \mu E^2 - KN^2 - c_\epsilon E_{tke}^{3/2} L^{-1}$$

with diffusivity $K \sim E_{tke}^{1/2} L$ to be used in ocean model, and diagnostic mixing length scale L

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- use IDEMIX in three global ocean models with identical initial and boundary conditions

	ICON-O (MPI-M)	FESOM (AWI)	MITgcm (M.Losch)
horizontal resolution	ca. 40 km	ca. 20–100 km	ca. 20–111 km
vertical levels	64	48	50
grid type	triangular	triangular	rectangular
grid staggering	C-grid	B-grid	C-grid

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- three different bottom forcing products (A,B,C) for IDEMIX
  experiments ICON-A, ICON-B, ICON-C, FESOM-A, .., , MITgcm-A, ...

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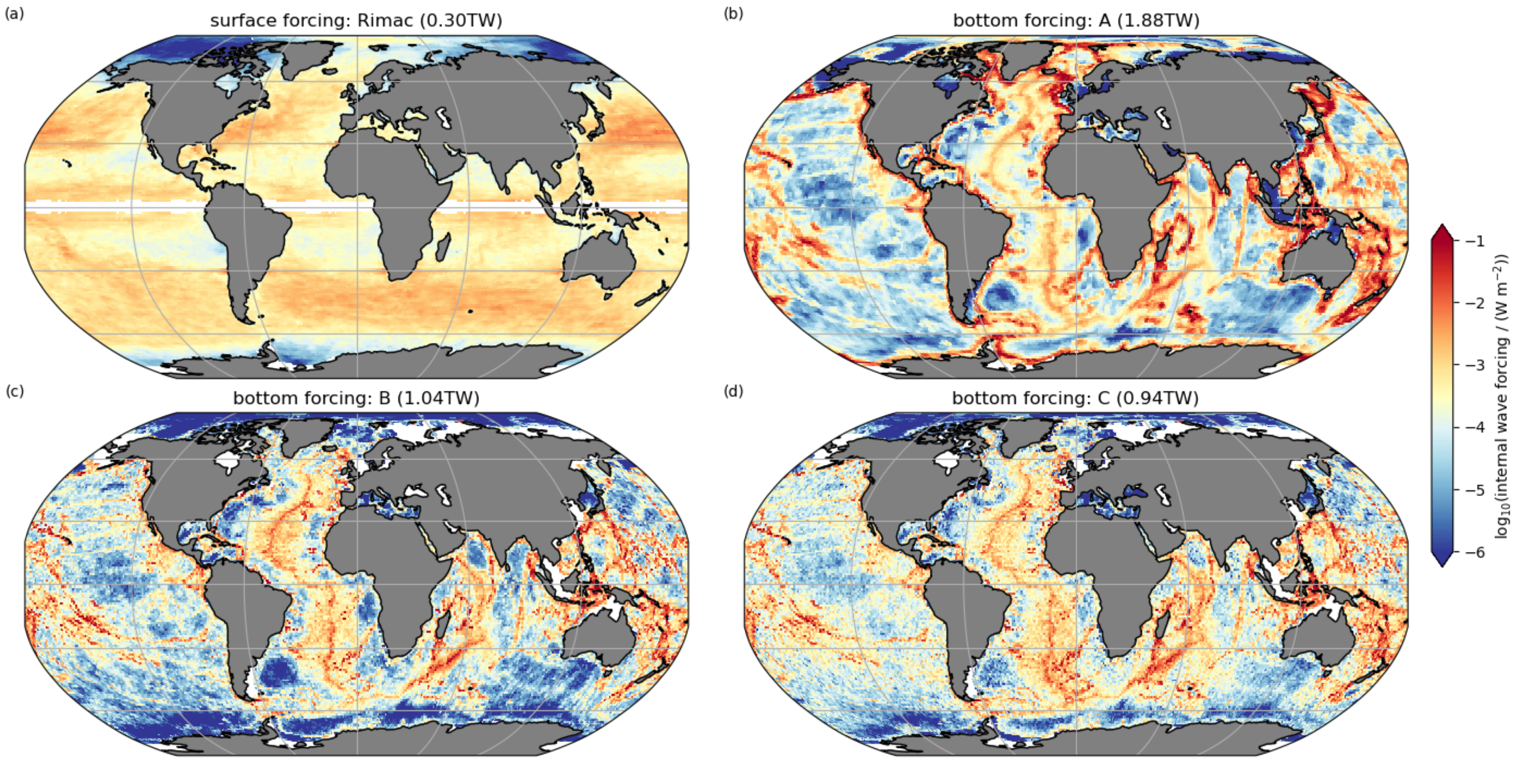
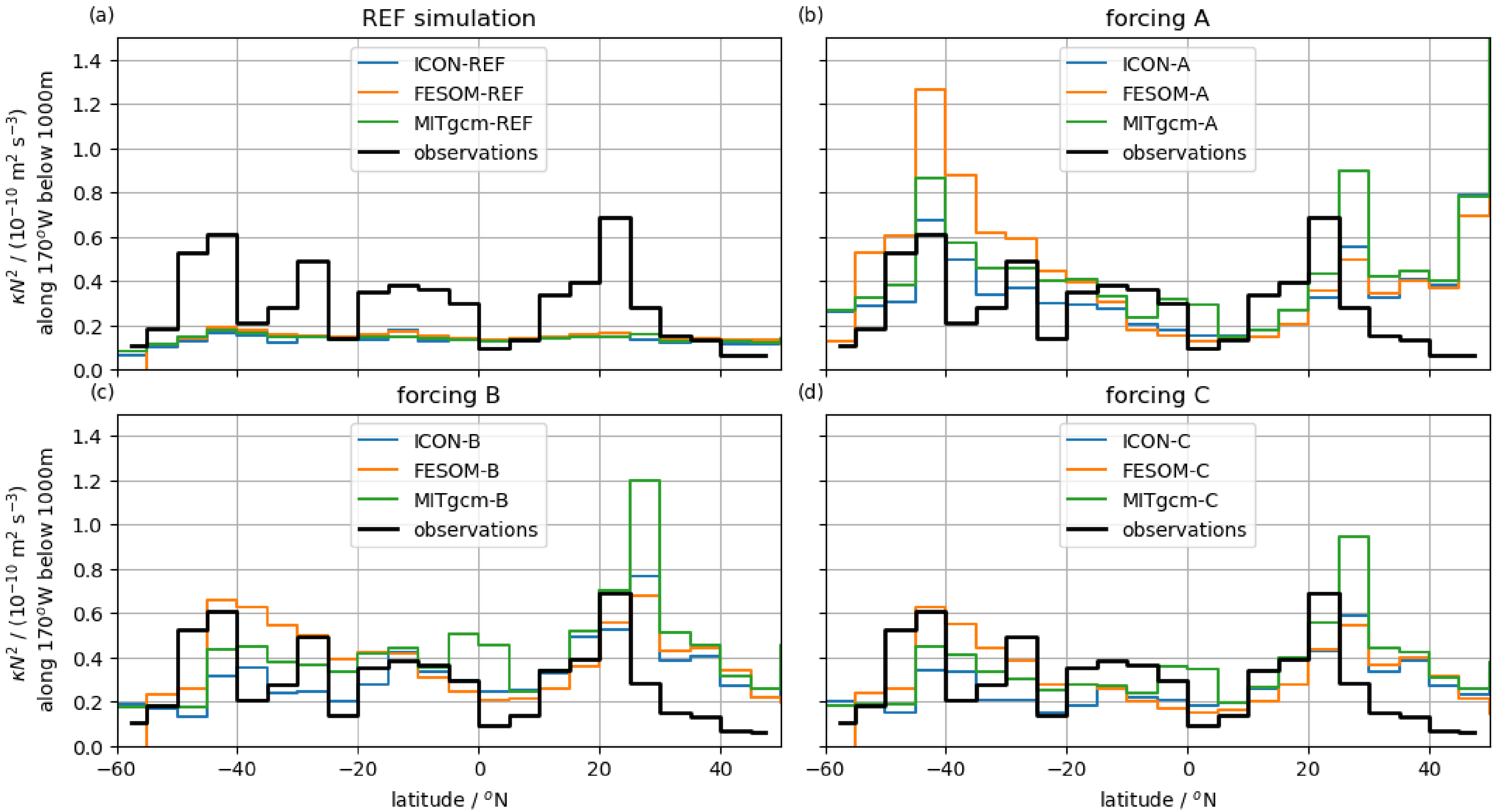
- reference experiments with artificial lower threshold for E_{tke}
  experiments ICON-REF, FESOM-REF, MITgcm-REF
- comparison allows to assess model-independent response to more realistic vertical mixing

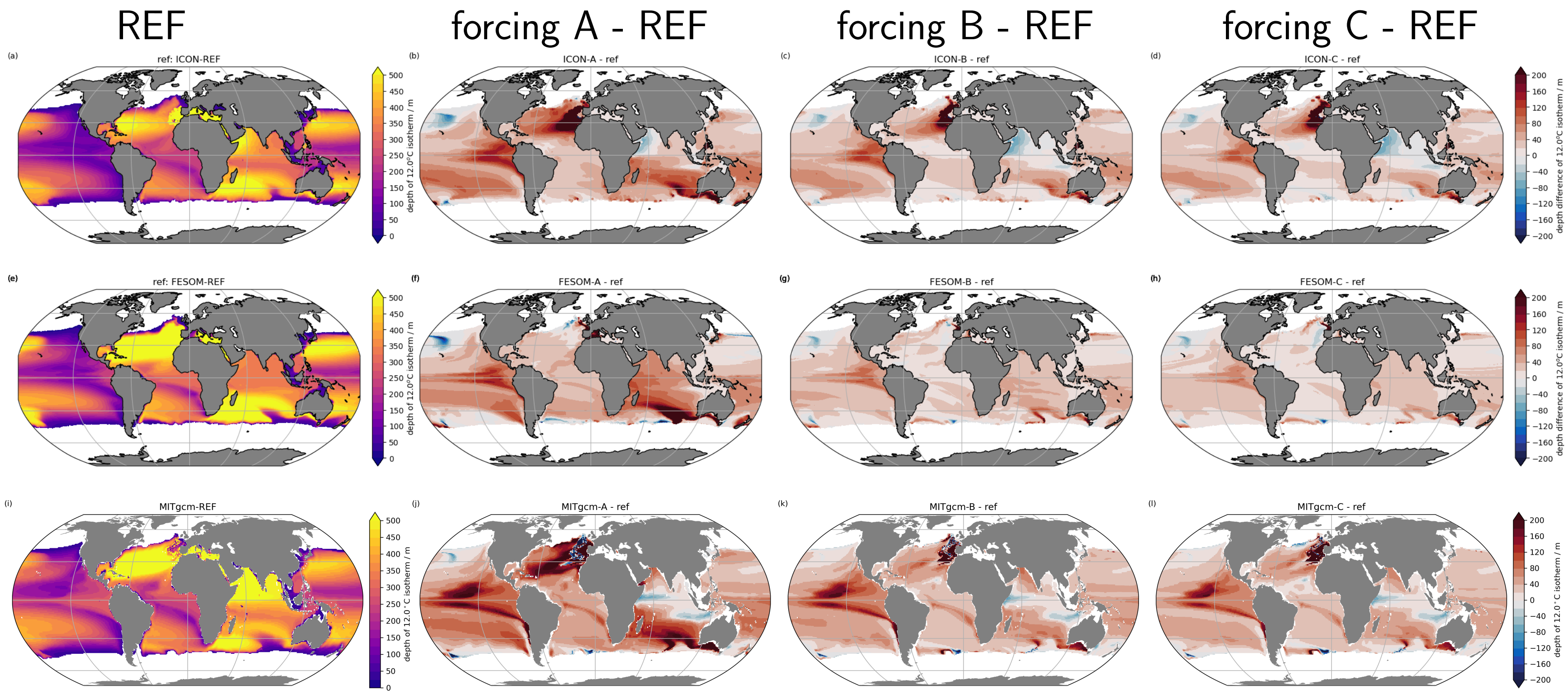
Nils Brüggemann, Martin Losch, Patrick Scholz, Friederike Pollmann, Sergey Danilov, Oliver Gutjahr, Johann Jungclaus, Nikolay Koldunov, Peter Korn, Dirk Olbers, and Carsten Eden, *Parameterized internal wave mixing in three ocean general circulation models*, (almost) submitted to JAMES (2023)



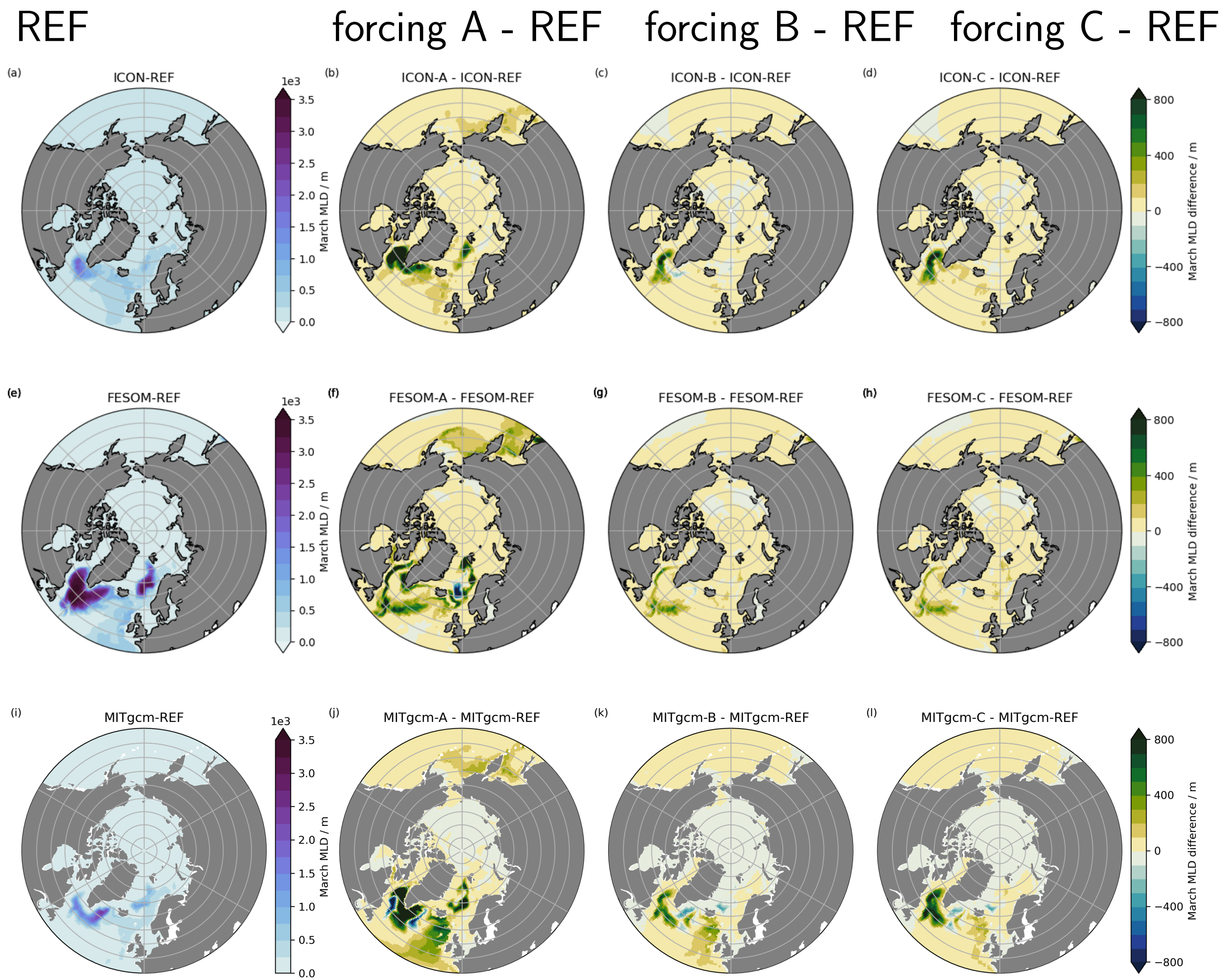
- surface forcing is smaller and kept the same in all experiments
- forcing A: 1.88 TW, from dissipation closure in barotropic tidal model
- forcing B: 1.04 TW, from linear theory (Bell, 1975)
- forcing C: 0.94 TW, from high-resolution ocean model with tides (STORMTIDE2)
- limitations and biases in all forcings → uncertainty



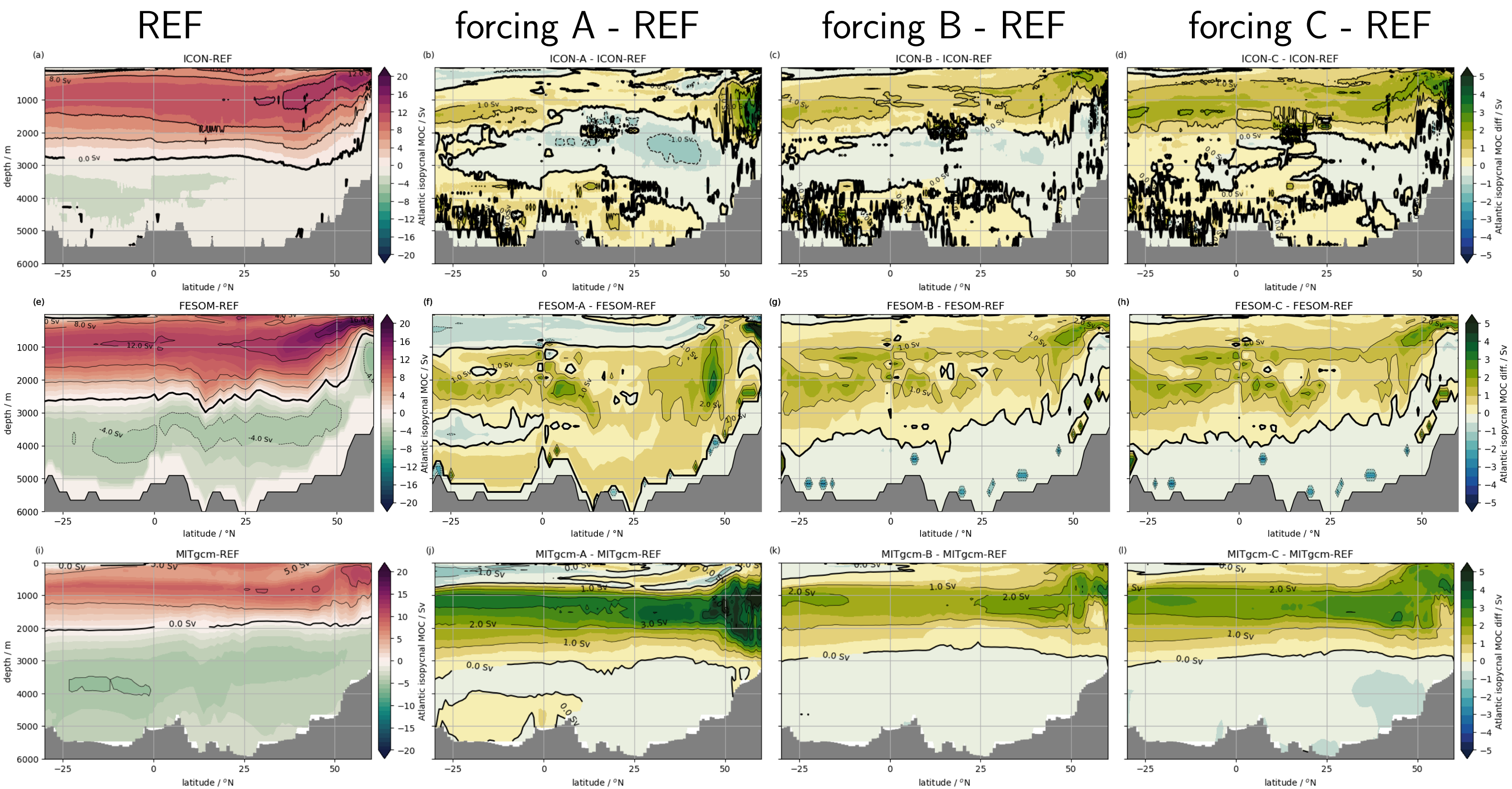
- comparison of mixing work KN^2 (average below $1000\,\mathrm{m}$) along 170^oW from observations, reference experiments and with IDEMIX
- large variations in models, but also large error (factor 2-3) in observations
- KN^2 with IDEMIX more realistic than in reference



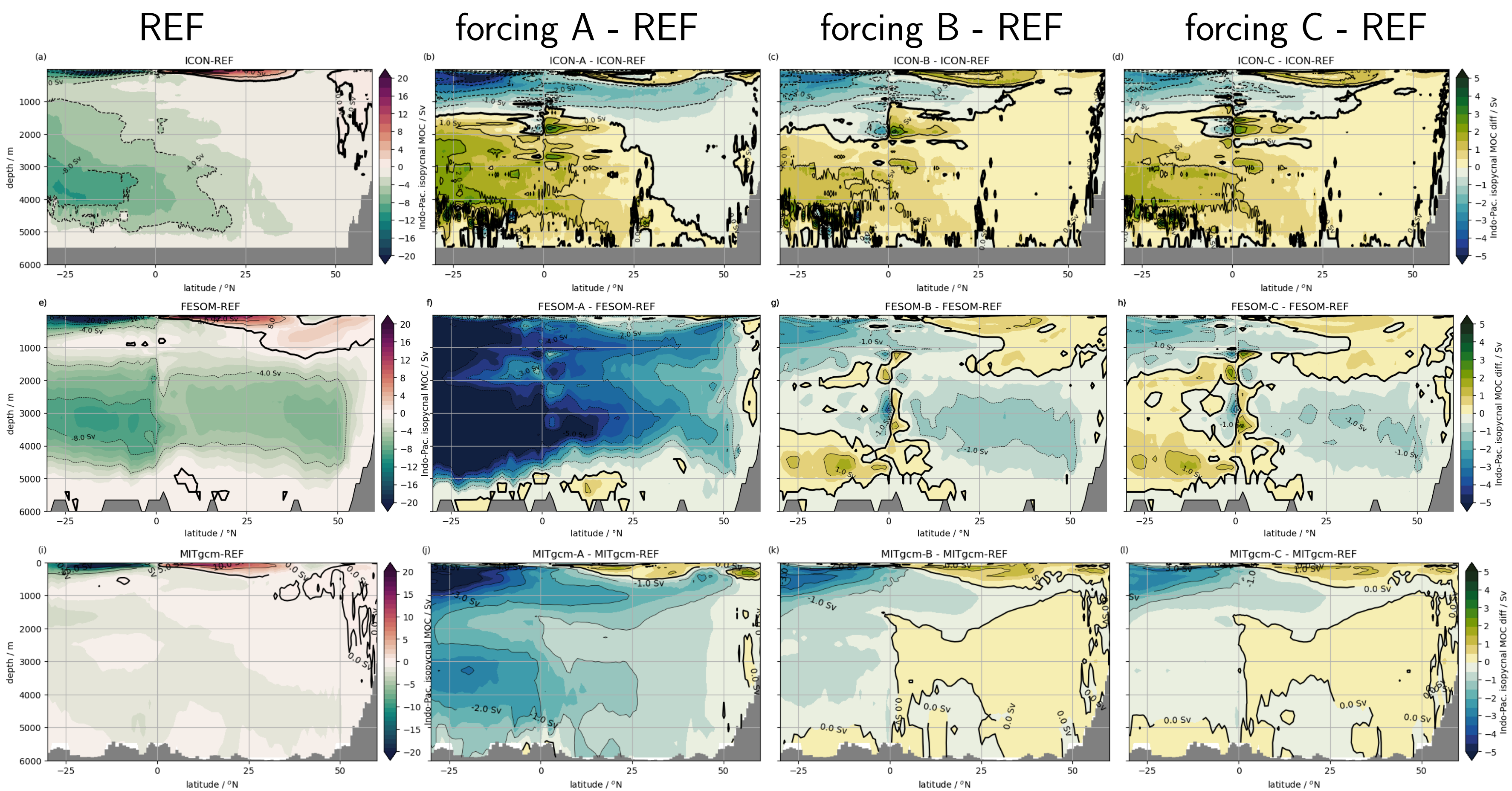
- thermocline depth (12^{o}C isotherm) in reference and difference using IDEMIX
- coherent change of thermocline
- mostly deeper thermocline with more mixing/IDEMIX



- mixed layer depth in reference and with IDEMIX
- coherent response of deeper mixed layer in subpolar North Atlantic, due to stronger preconditioning for convection



- (residual mean) meridional overturning streamfunction in Atlantic in reference/ with IDEMIX
- coherent response of stronger upper (wind-driven) cell in relation to deeper convection depths (we do not know the reason for this relation)
- incoherent response in lower (mixing-driven) cell



- meridional overturning streamfunction in Indo-Pacific in reference/ with IDEMIX
- coherent response of stronger (wind-driven) shallow overturning cells due to larger surface area of ventilated density layers
- incoherent model response in (mixing-driven) bottom cell
- excessive transports in FESOM-A due to too deep convection depth in Wedell Sea

Summary

- IDEMIX concept reduces dimensions of radiative wave energy transport equation
- used for vertical mixing in three global ocean models with three tidal forcing functions
- larger and more realistic mixing work KN^2 with IDEMIX
- one forcing function overestimates KN^2 in Southern Ocean

Summary

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Coherent model response :

- deeper thermocline depth
- increase of (wind-driven) shallow overturning cells in Indo-Pacific
- deeper mixed layer depths in subpolar North Atlantic
- increase of (wind-driven) upper cell of Atlantic overturning circulation
- increase in northward heat transport in Atlantic

Summary

- IDEMIX concept reduces dimensions of radiative wave energy transport equation
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Coherent model response :

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- increase of (wind-driven) shallow overturning cells in Indo-Pacific
- deeper mixed layer depths in subpolar North Atlantic
- increase of (wind-driven) upper cell of Atlantic overturning circulation
- increase in northward heat transport in Atlantic

- incoherent model response in (mixing-driven) lower cell in Atlantic and Indo-Pacific
- due to excessive numerical mixing?

Brüggemann et al: *Parameterized internal wave mixing in three ocean general circulation models*, (almost) submitted to JAMES (2023)