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Atmospheric gravity waves and their influence on weather and climate: Challenges and new approaches

**U. Achatz, G.S. Völker, Y.-H. Kim, G. Bölöni, J. Muraschko, M. Fruman (all GU Frankfurt),
R. Klein (FU Berlin) & ...**



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Atmospheric Gravity Waves: Impacts & Issues



Atmospheric GWs: Impacts and issues

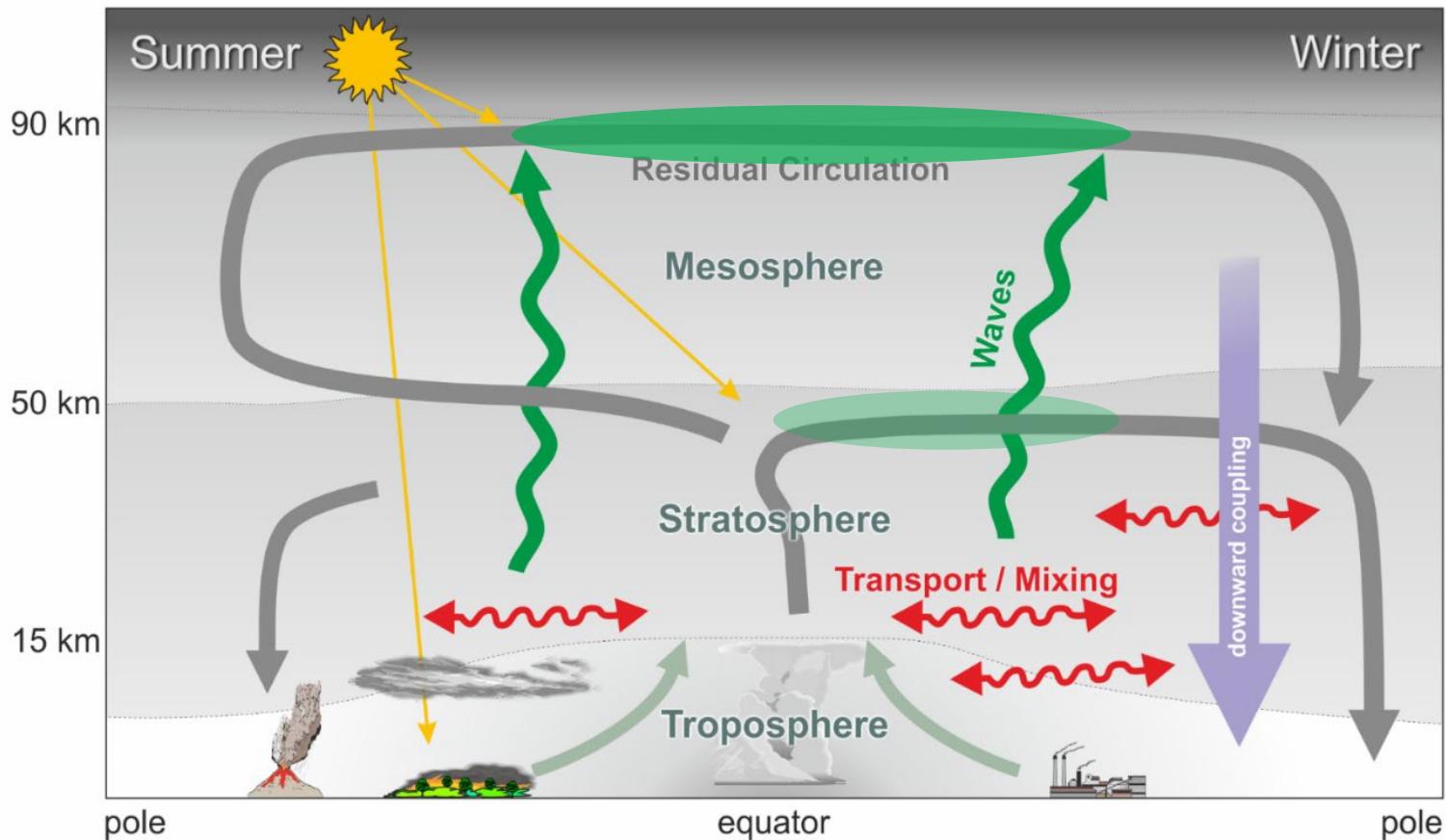


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- Rossby-wave and **GW breaking** force the Brewer-Dobson circulation



Atmospheric GWs: Impacts and issues

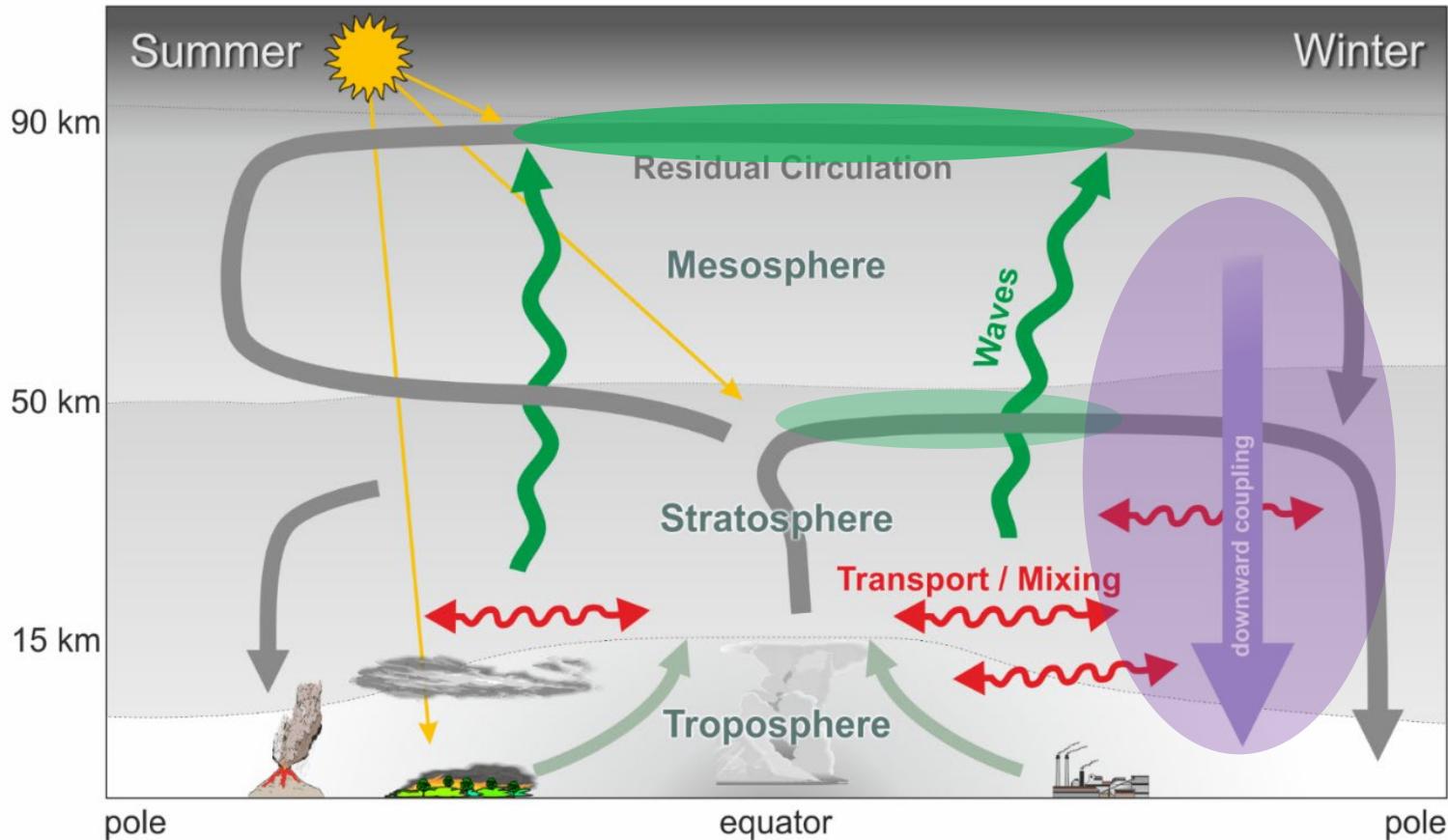


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- **Downward control:** Middle atmosphere influences tropospheric climate (Haynes et al 1991)



Atmospheric GWs: Impacts and issues

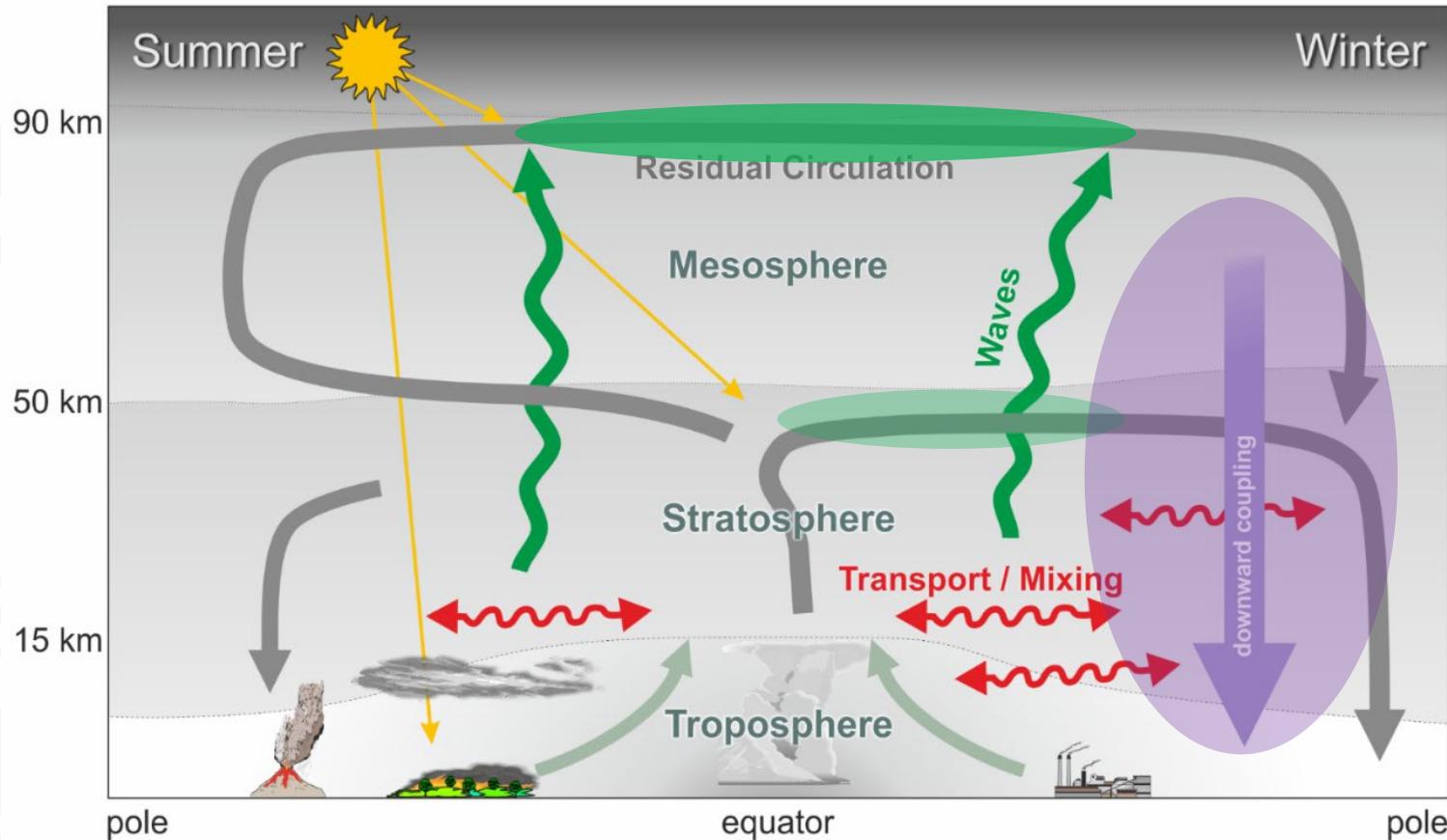


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- Rossby-wave and **GW breaking** force the Brewer-Dobson circulation
- **Downward control:** Middle atmosphere influences tropospheric climate (Haynes et al 1991)
- BDC under **climate change:** Impact from GWs (Butchard 2014)



Atmospheric GWs: Impacts and issues

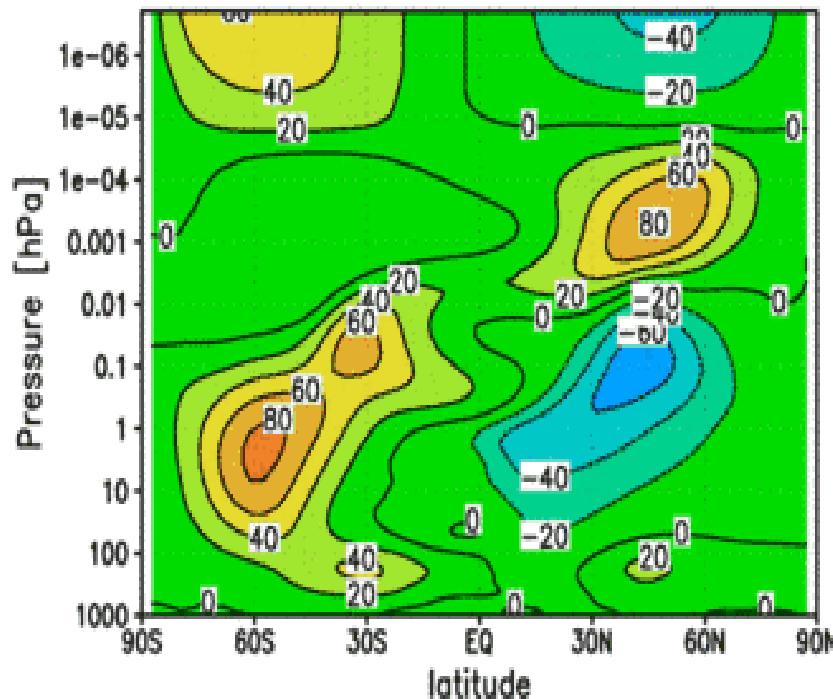


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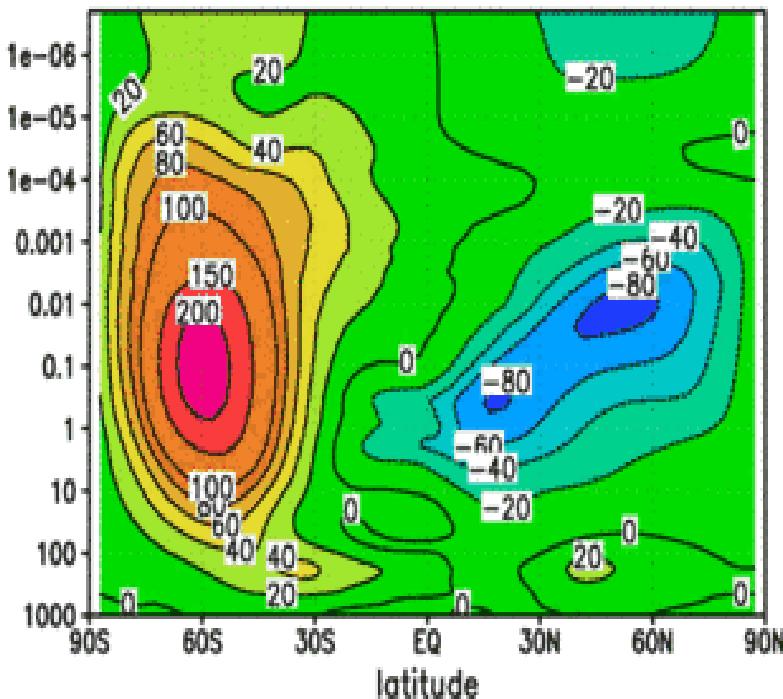
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Zonal-mean zonal winds (westerlies)



With GW parameterization



Without GW parameterization

Schmidt et al (2006)

Atmospheric GWs: Impacts and issues

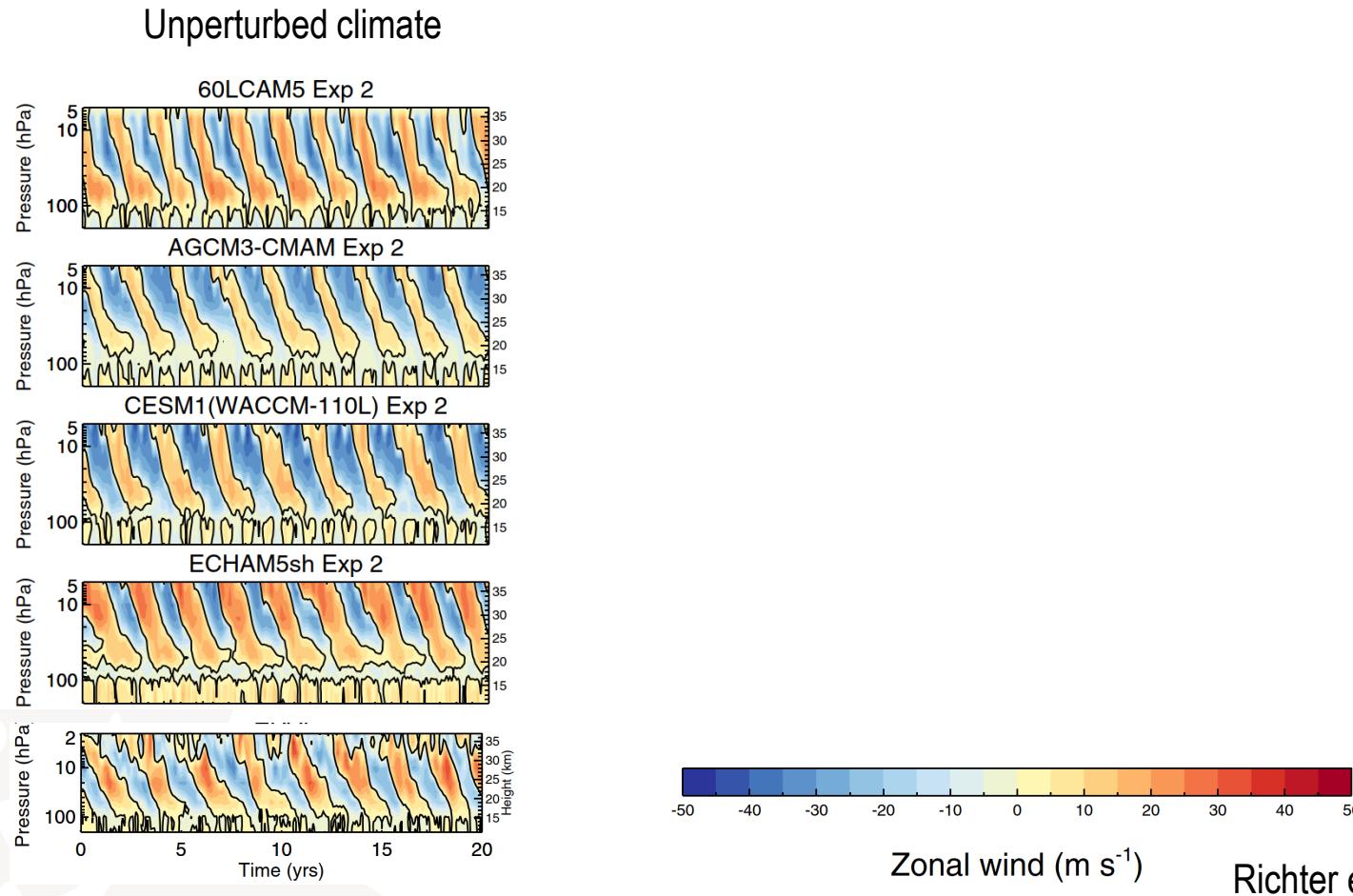


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Quasi-Biennial Oscillation (QBO) of zonal-mean zonal wind over the equator:



Atmospheric GWs: Impacts and issues

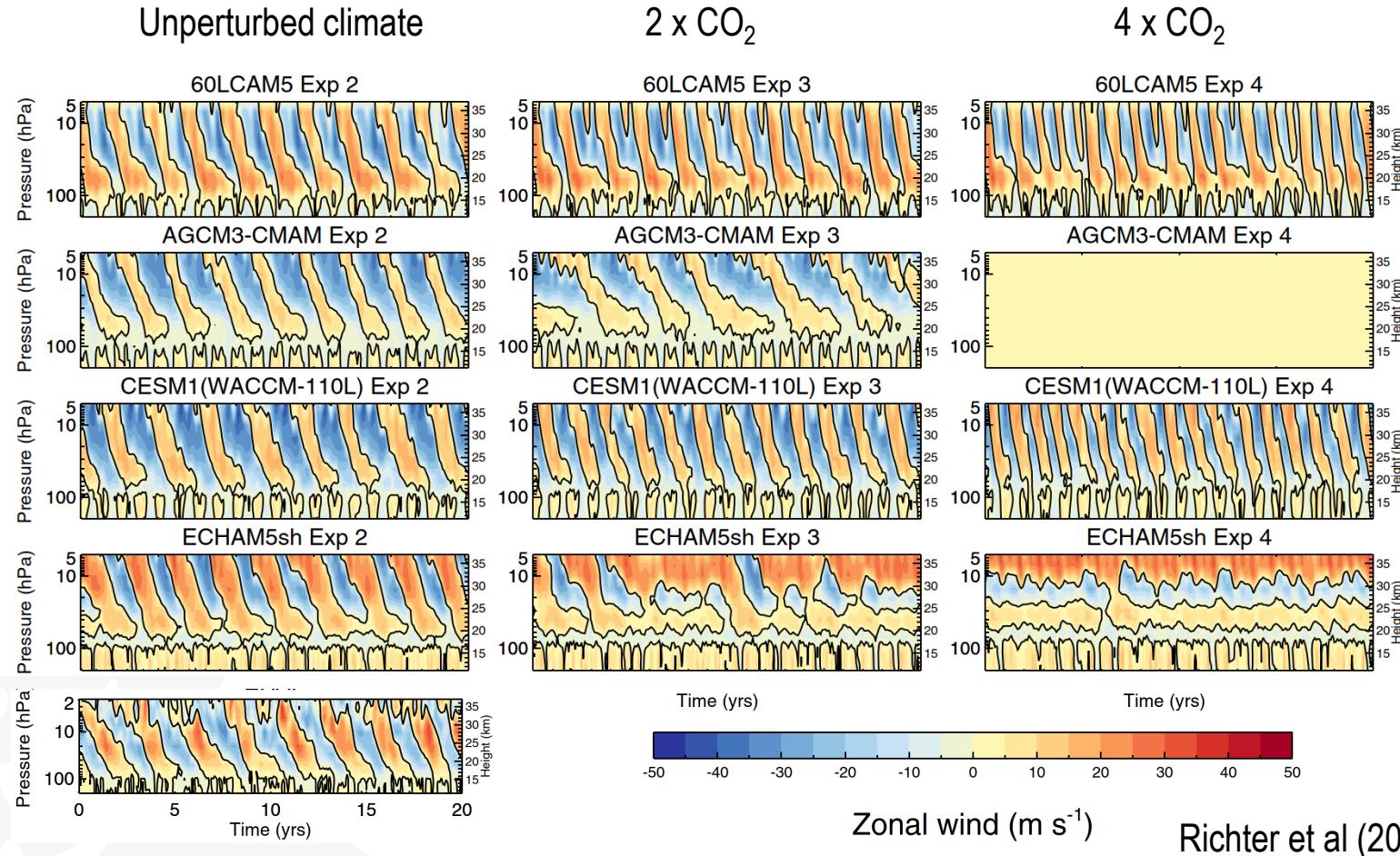


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Quasi-Biennial Oscillation (QBO) of zonal-mean zonal wind over the equator:
Model predictions for QBO under climate change (Schirber et al 2014, Richter et al 2020)



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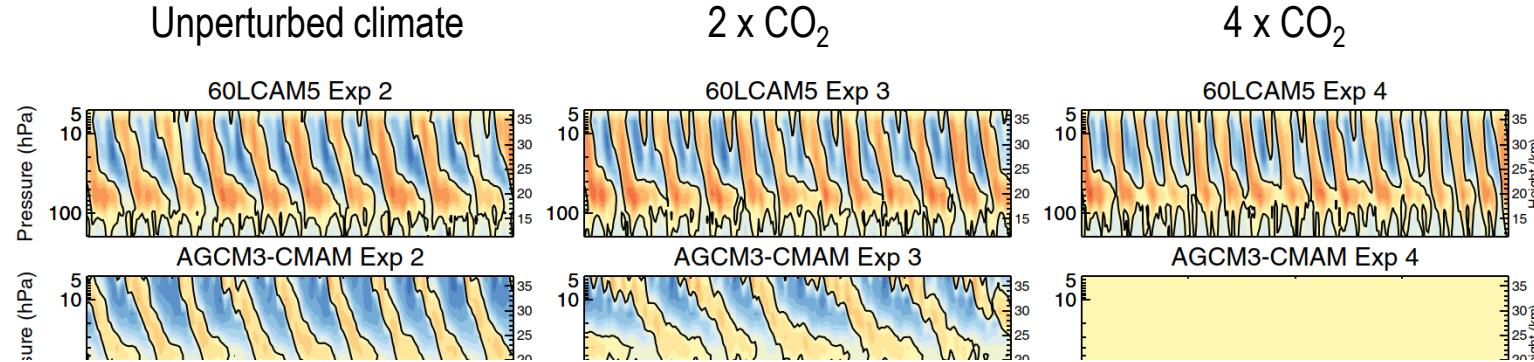


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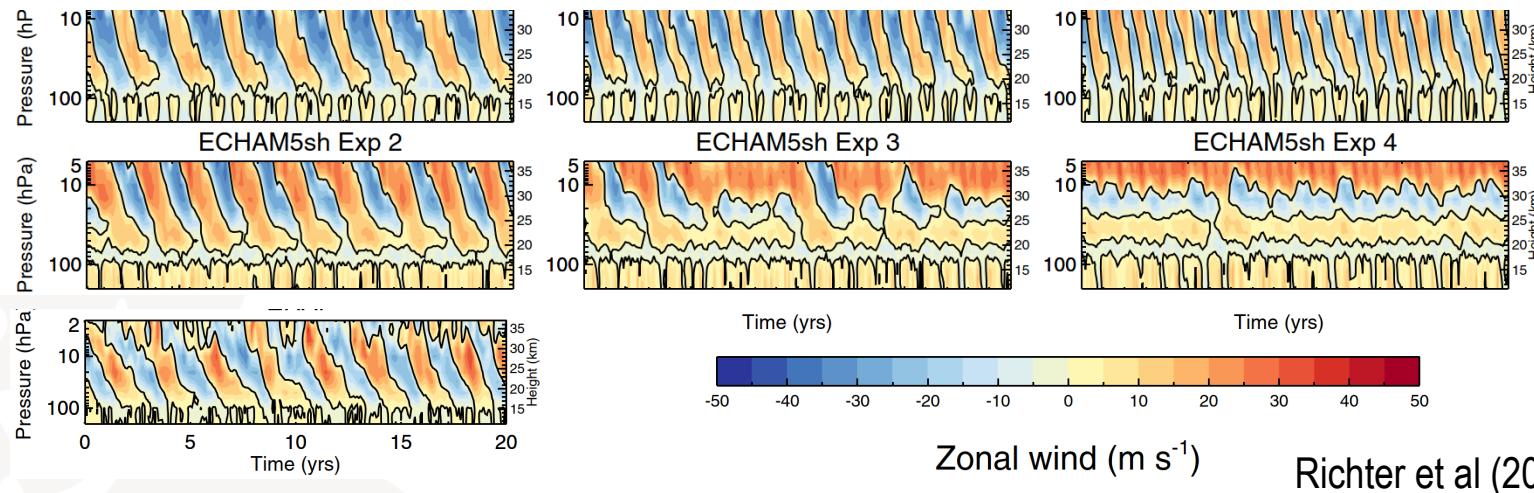
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Quasi-Biennial Oscillation (QBO) of zonal-mean zonal wind over the equator:
Model predictions for QBO under climate change (Schirber et al 2014, Richter et al 2020)



Significant dependence on choice of implemented GW parameterization



Richter et al (2020)

Atmospheric GWs: Impacts and issues



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Try and resolve everything?

1.4 km explicitly simulated convection
2018110100+36h (h3f7)



Verifying satellite image
MSG obs 2018020112



Atmospheric GWs: Impacts and issues



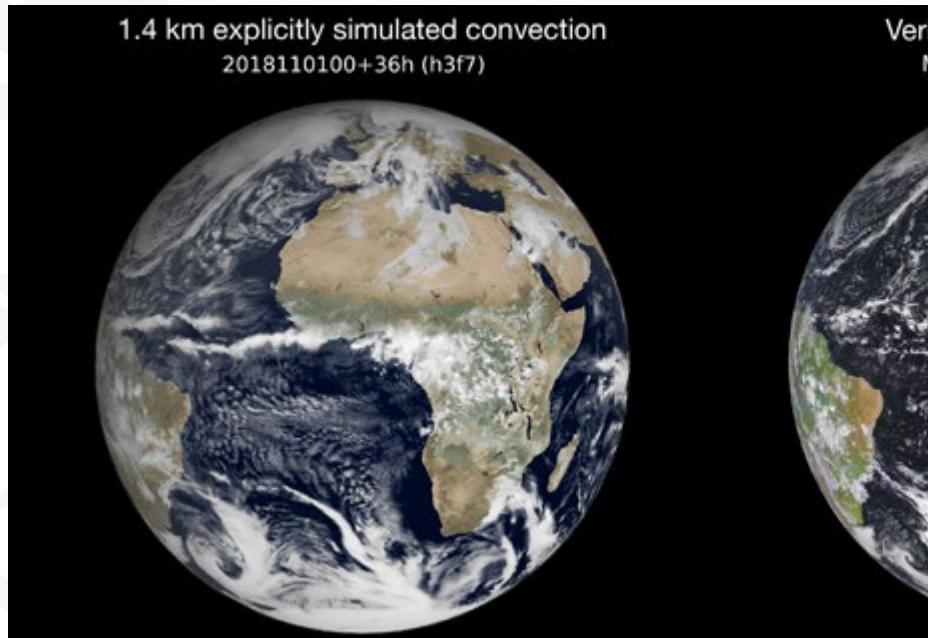
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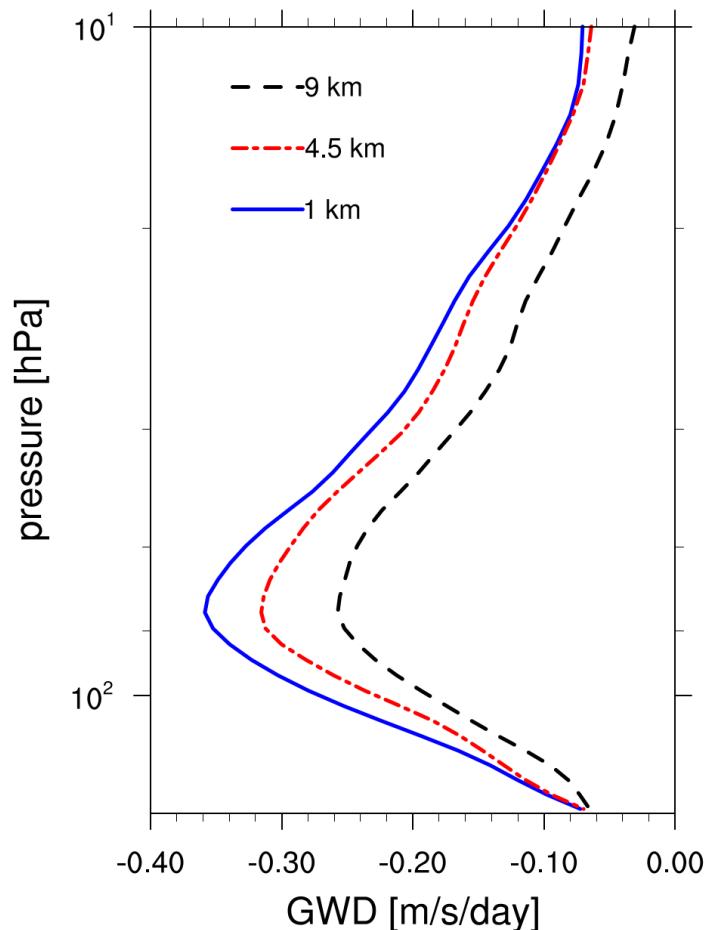
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Try and resolve everything?

- **No convergence** at accessible resolutions
(e.g. Polichtchouk et al 2022)



GWD $20^{\circ}\text{N} - 80^{\circ}\text{N}$ (Polichtchouk et al 2022)



Atmospheric GWs: Impacts and issues



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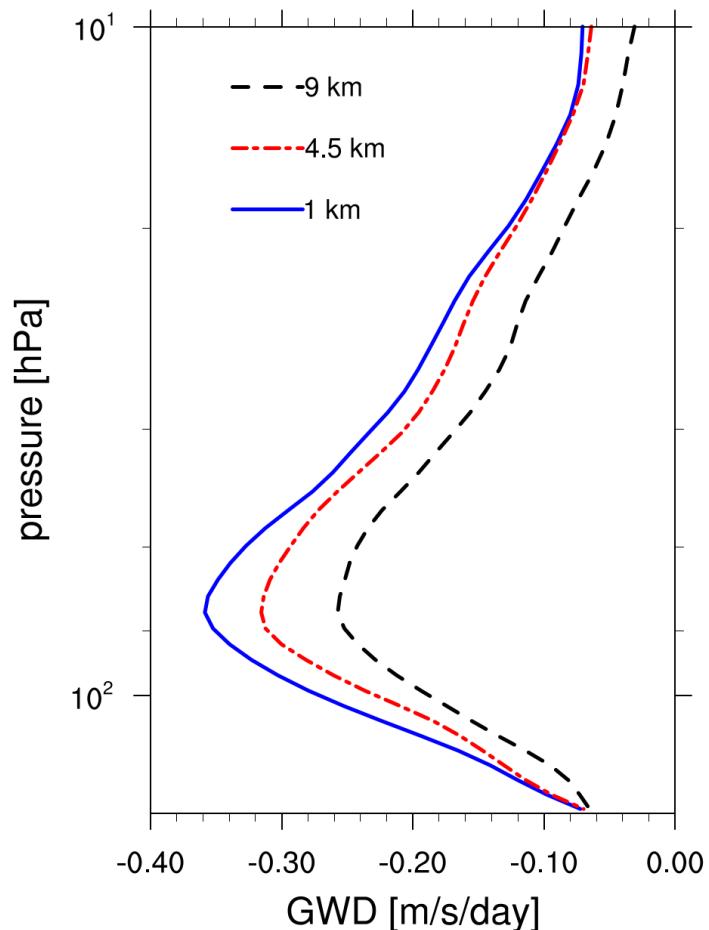
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Try and resolve everything?

- **No convergence** at accessible resolutions (e.g. Polichtchouk et al 2022)
- **Model hierarchy** needed for conceptual understanding (Held 2005)



GWD $20^{\circ}\text{N} - 80^{\circ}\text{N}$ (Polichtchouk et al 2022)





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GW-Mean-Flow Interaction: Theory in a Nutshell

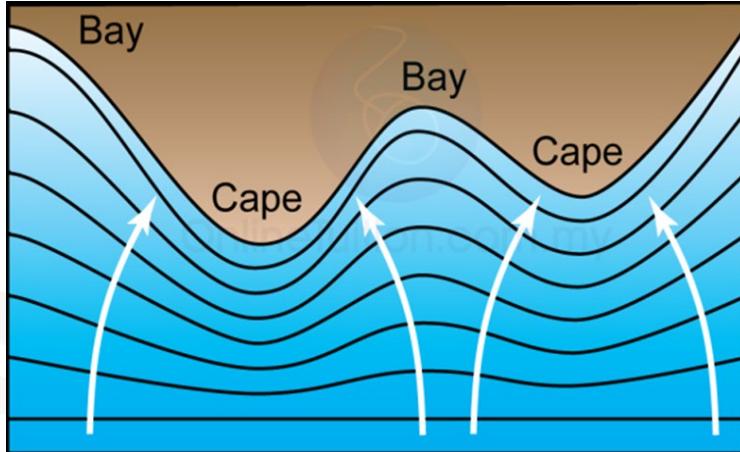
Theory: GW-mean-flow interaction



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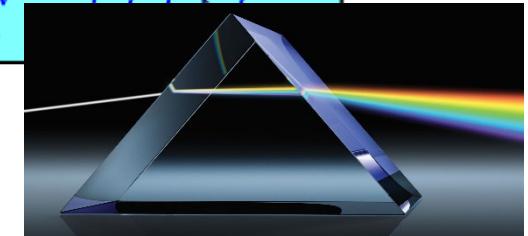
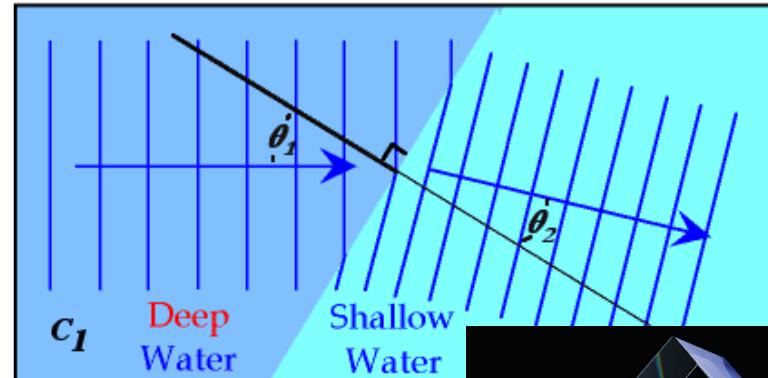
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Short-wavelength GWs in the **atmosphere**
related to **geometric optics**:

- Atmosphere = medium with refractive properties
- GWs modify the medium
- Two-way interaction described using WKB theory



Theory: GW-mean-flow interaction



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WKB theory for

- GWs (locally monochromatic or weak amplitude)
- on a slowly varying stratified mean flow $\mathbf{u}(x, t)$

(e.g. Bretherton 1966, Grimshaw 1975, Achatz et al 2017, Achatz 2022):

- frequency and wave number connected by dispersion relation

$$\omega = \Omega(\mathbf{x}, \mathbf{k}, t) = \mathbf{k} \cdot \mathbf{u}(\mathbf{x}, t) \pm \underbrace{\sqrt{\frac{N^2(z)k_h^2 + f^2(\phi)k_z^2}{k_h^2 + k_z^2}}}_{\hat{\omega} \text{ (intrinsic frequency)}}$$

- GW amplitudes connected by polarization relations, e.g. ($\hat{b} = g \hat{\theta}/\bar{\theta}$)

$$\hat{\mathbf{u}} = -\frac{i}{mN} \frac{N^2 - \hat{\omega}^2}{\hat{\omega}^2 - f^2} (\mathbf{k}_h \hat{\omega} - if \mathbf{e}_z \times \mathbf{k}_h) \hat{b}$$

$$\hat{w} = \frac{i\hat{\omega}}{N^2} \hat{b}$$

Theory: GW-mean-flow interaction



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Prognostic equations for weak-amplitude GWs:

Spectral wave-action linked to spectral energy density via

$$\mathcal{N}(x, k, t) = \mathcal{E}(x, k, t)/\hat{\omega}$$

and satisfies

$$(\partial_t + \mathbf{c}_g \cdot \nabla_x + \dot{\mathbf{k}} \cdot \nabla_k) \mathcal{N} = D + S \quad \mathbf{c}_g = \nabla_k \Omega \quad \dot{\mathbf{k}} = -\nabla_x \Omega$$

D = GW sources and sinks (e.g. wave breaking)

S = scattering due to GW-GW interactions or interactions with mesoscale balanced motion

- theory from oceanography (Hasselmann 1966, Eden et al 2019) assumes weak mean flows
- neglected in the atmosphere

If $D = S = 0$ then \mathcal{N} is conserved along rays $(x, k)(t)$ satisfying

$$d_t x = \mathbf{c}_g \quad d_t \mathbf{k} = \dot{\mathbf{k}}$$

Theory: GW-mean-flow interaction



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GW impact on (phase averaged)mean flow, with fluxes from polarization relations:

Horizontal momentum

$$D_t \mathbf{u} = \cdots - \frac{1}{\bar{\rho}} \nabla_x \cdot \langle \bar{\rho} \mathbf{v}' \mathbf{u}' \rangle \quad \langle \bar{\rho} \mathbf{v}' \mathbf{u}' \rangle = \bar{\rho} \int d^3 k \overleftrightarrow{\mathbf{M}}(\mathbf{k}) \mathcal{N}$$

Entropy

$$D_t \theta = \cdots - \nabla_x \cdot \langle \mathbf{u}' \theta' \rangle \quad \langle \mathbf{u}' \theta' \rangle = \int d^3 k \Theta(\mathbf{k}) \mathcal{N}$$

More conventional (pseudomomentum) approach is

$$D_t \mathbf{u} = \cdots - \frac{1}{\bar{\rho}} \nabla_x \cdot \int d^3 k \mathbf{c}_g \mathbf{k}_h \mathcal{N} \quad D_t \theta = \cdots - 0$$

but this can lead to errors (Wei et al 2019)

Non-acceleration theorem (Charney & Drazin 1961, Andrews & McIntyre 1978, Achatz 2022):

GWs have no impact on a synoptic-scale mean flow if

- they are steady,
- their spatial distribution is horizontally homogeneous, and
- there are no GW sources and sinks (and no wave-wave interactions)



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Numerical Approach: Lagrangian Ray Tracing **MS-GWaM**

Non-acceleration theorem (Charney & Drazin 1961, Andrews & McIntyre 1978, Achatz 2022):

GWs have no impact on a synoptic-scale mean flow if

- they are **steady**,
- their spatial distribution is **horizontally homogeneous**, and
- there are **no GW sources and sinks** (and no wave-wave interactions)

Conventional GW parameterizations:

- Steady state (no transience)
Equilibrium profiles assumed, i.e. instantaneous propagation from source to model top
- Single column (1D):
no horizontal GW propagation, no horizontal variations of GW energy taken into account
- Rely exclusively on GW breaking

MS-GWaM (Multi-Scale Gravity-Wave Model):

All three processes taken into account

Numerics: Conservative Lagrangian Numerics



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Without sources and sinks:

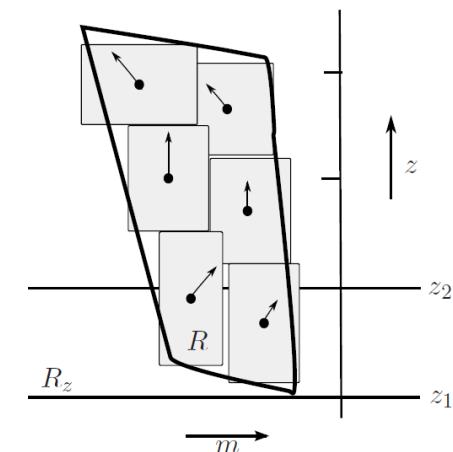
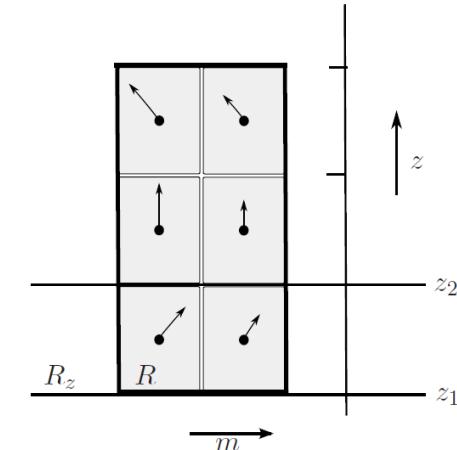
$$(\partial_t + \mathbf{c}_g \cdot \nabla_x + \dot{\mathbf{k}} \cdot \nabla_k) \mathcal{N} = 0$$

Phase-space velocity nondivergent ($\nabla_x \cdot \mathbf{c}_g + \nabla_k \cdot \dot{\mathbf{k}} = 0$)

⇒ flow is **volume preserving**

Region of nonzero \mathcal{N} approximated by **rectangular ray volumes**

- Ray volumes move with **central ray**
- Ray volumes **change spatial and wavenumber extent** in area-preserving manner



Numerics: Conservative Lagrangian Numerics

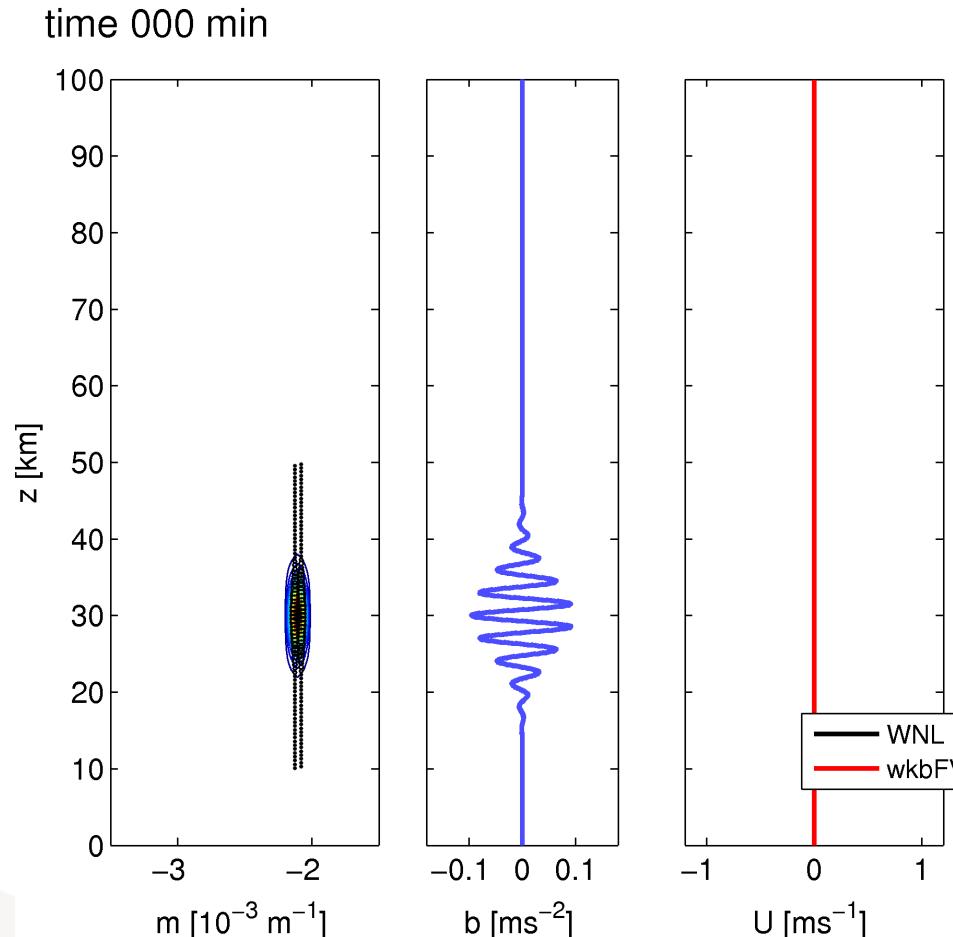


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GW packet in Boussinesq flow:
1D, i.e. GW energy and mean flow horizontally homogeneous



Numerics: MS-GWaM in UA-ICON



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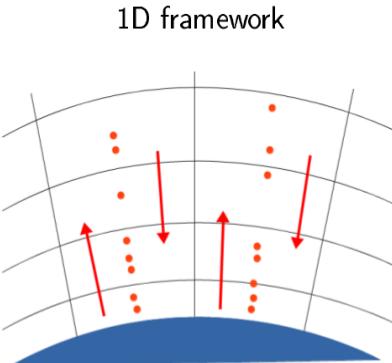
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MS-GWaM in the
Upper-Atmosphere extension of ICON (DWD/MPI)

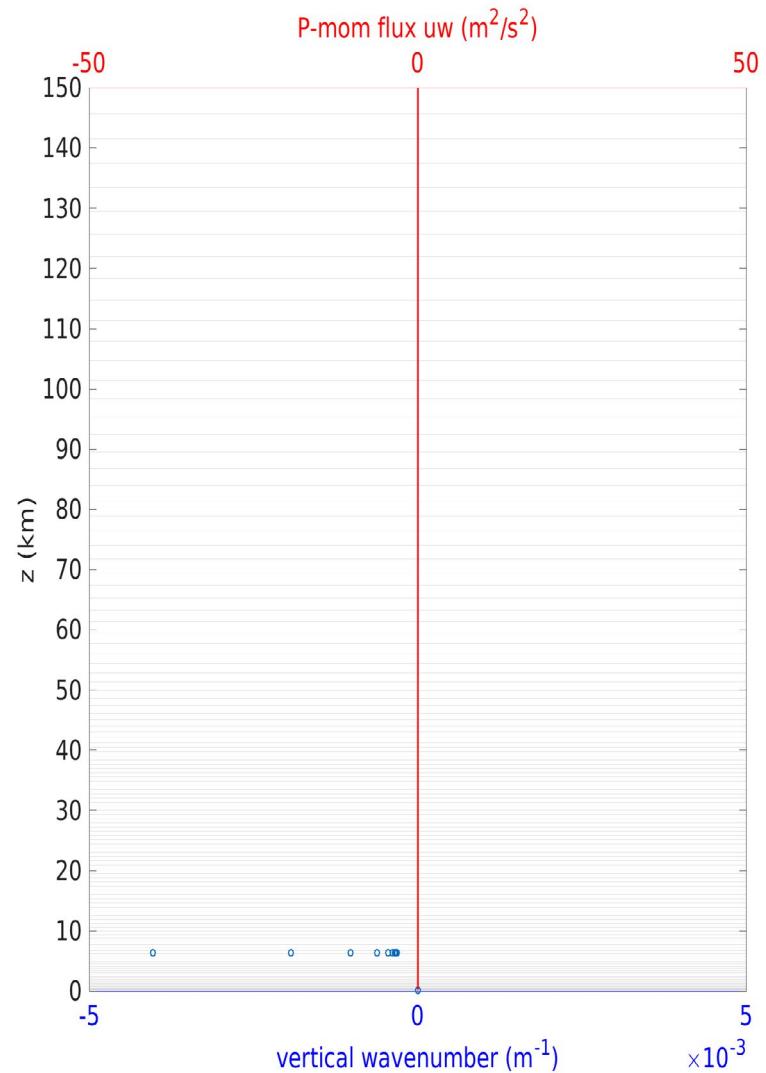
$\Delta x \sim 160$ km (R2B4), $\Delta z \sim 1$ km, $z_{top} = 150$ km



UA-ICON
(Borchert et al 2019)



Fits well to the current MPI
communicator



Numerics: MS-GWaM in UA-ICON



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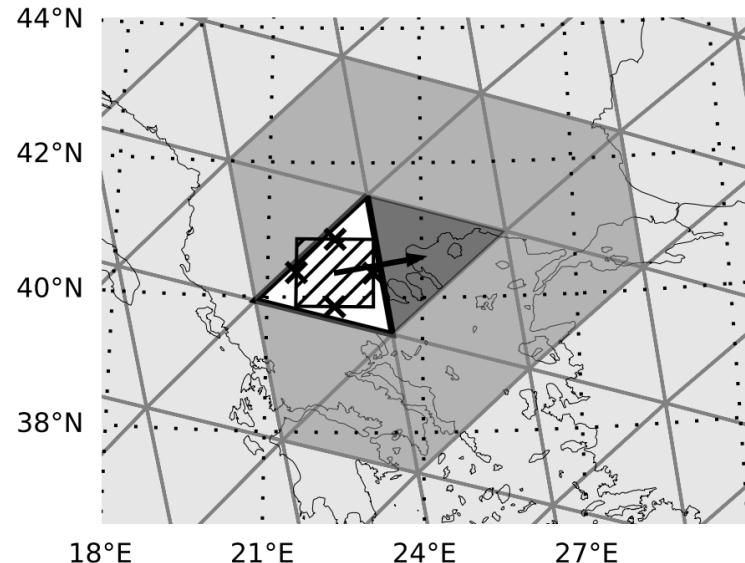
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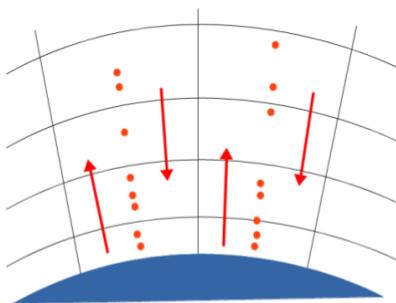
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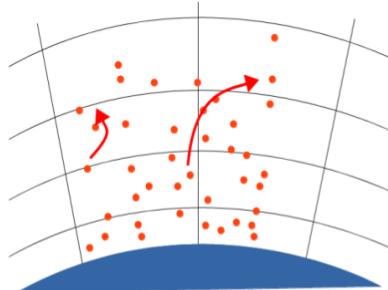


1D framework



Fits well to the current MPI
communicator

3D framework



Requires new MPI communication style
for Lagrangian particles

Implementation 3D

- Cell-based ray-volume handling
- Handover from cell to cell
- MPI parallelization

Sources:

- GWs from **convection** (based on Choi et al 2011)
 - using latent heat release
 - small-scale waves from convective cells
 - larger-scale waves from mesoscale convective systems
- **Background GW source** based on Orr et al (2010)
 - Seasonal dependence
 - In each hemisphere horizontally homogeneous
- **Orographic GWs parameterized outside MS-GWaM** (Lott and Miller 1997)

Sink:

Wave breaking due to static instability, using **saturation approach**
(Lindzen 1981, Böloni et al 2016)



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Effects of GW Transience on Momentum-Flux Intermittency

Effects of GW transience: GW momentum-flux intermittency

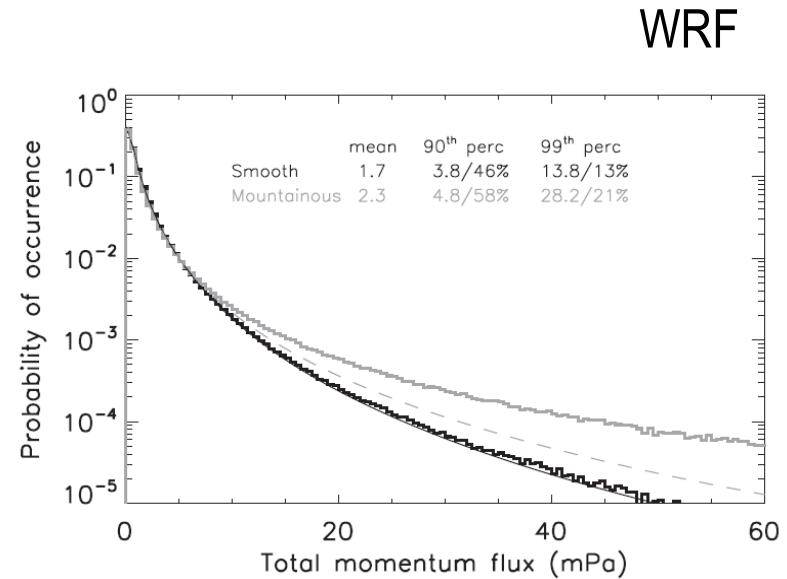
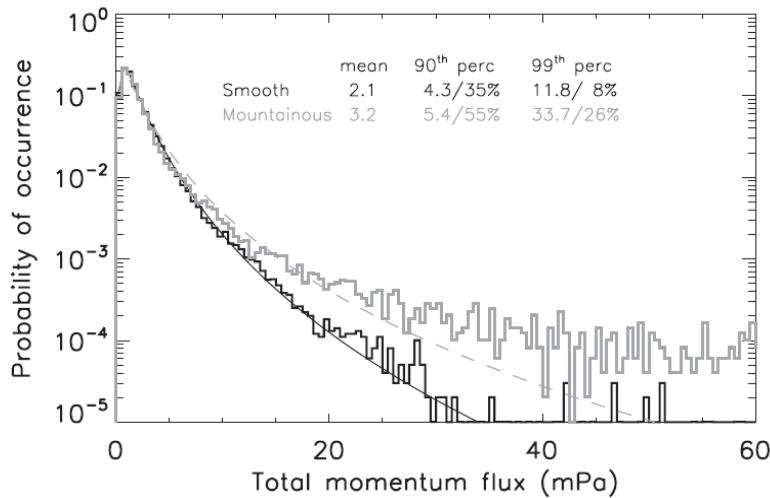


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Hertzog et al (2012): Vorcore measurements



Effects of GW transience: GW momentum-flux intermittency

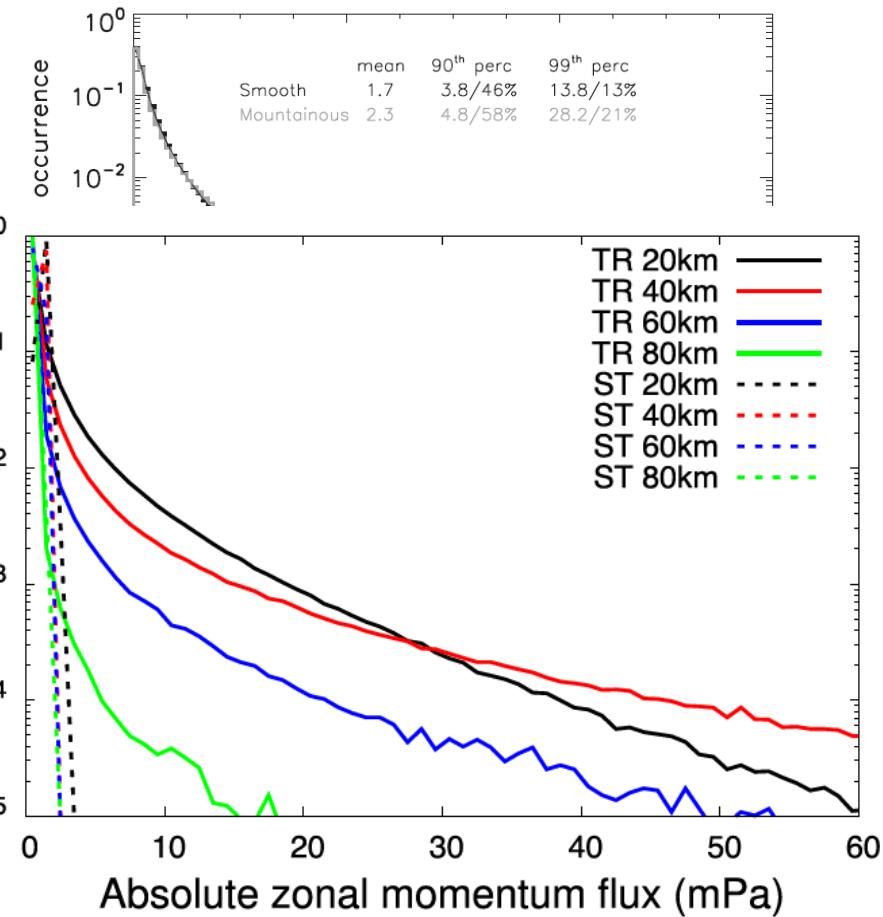
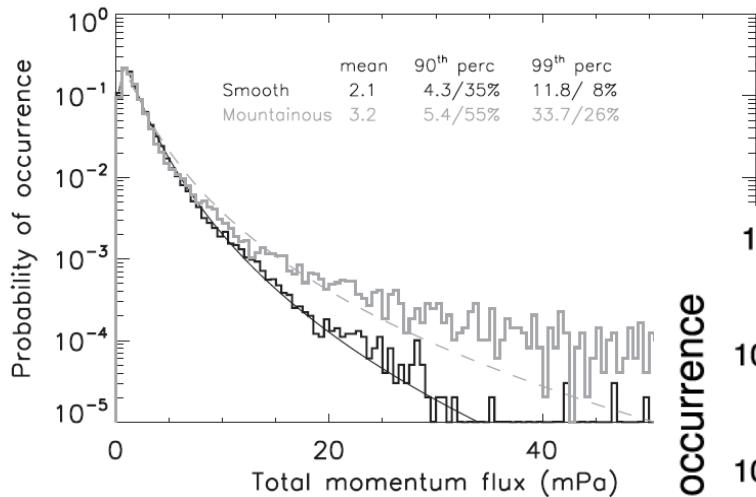


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Hertzog et al (2012): Vorcore measurements



Bölöni et al (2021): 65°S – 50°S in Oct

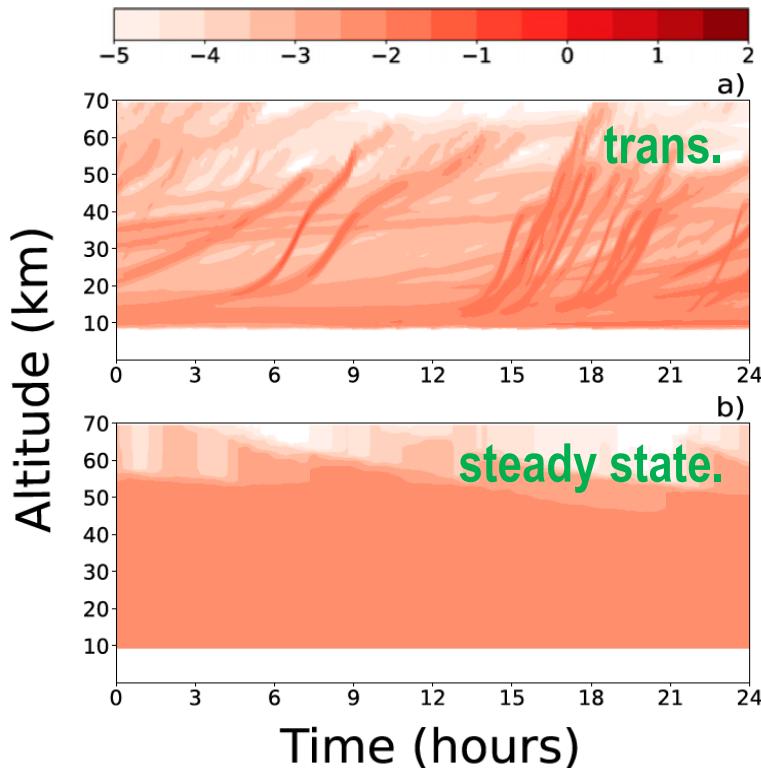
Effects of GW transience: GW momentum-flux intermittency



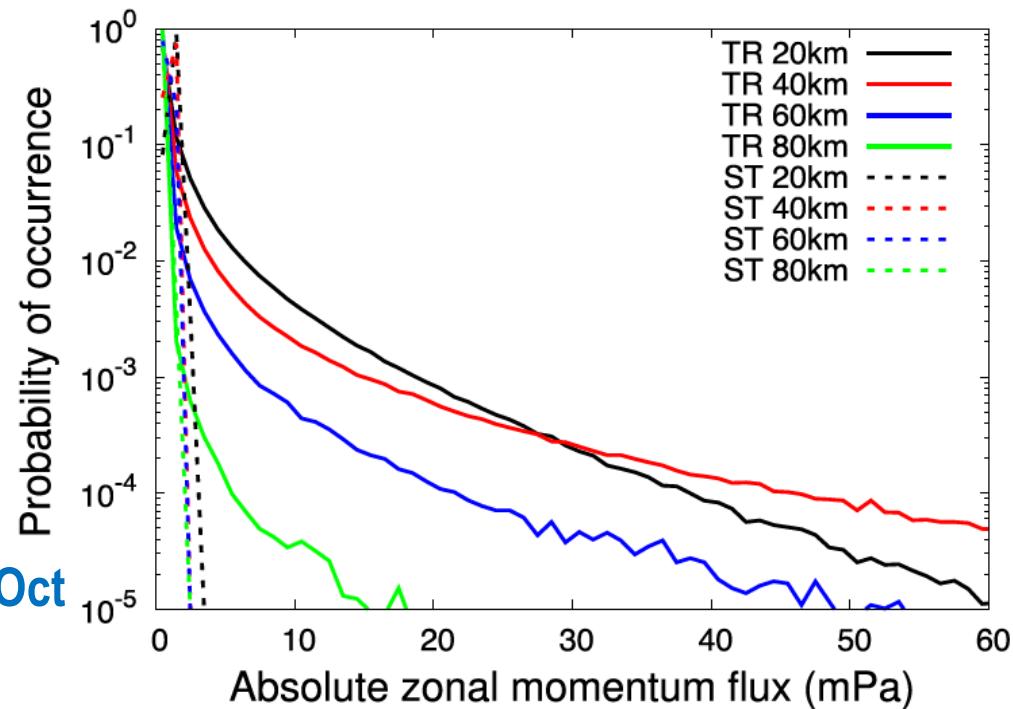
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(150°W, 60°S) on Jun 1st 1998



Bölöni et al (2021): 65°S – 50°S in Oct

Effects of GW transience on GW momentum-flux intermittency



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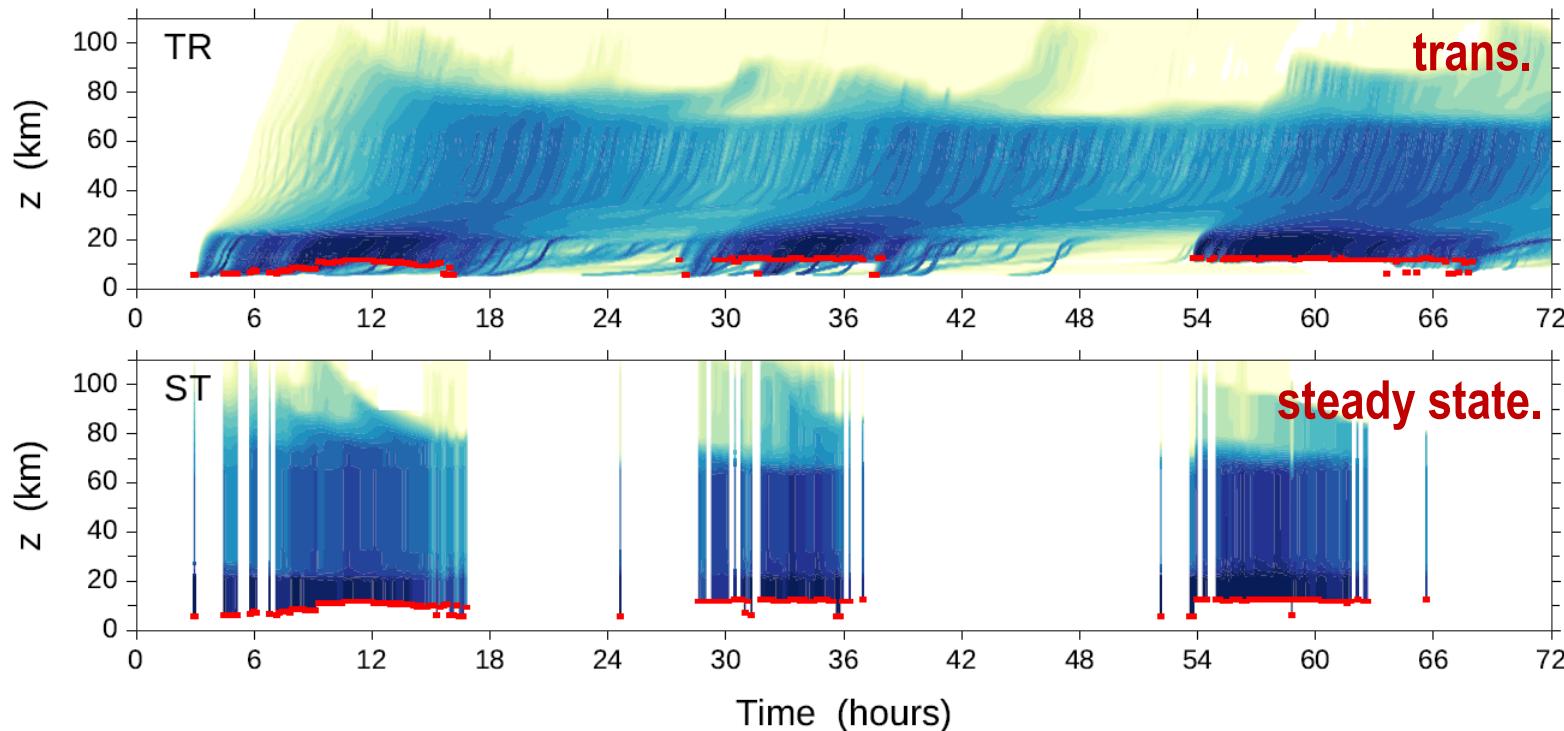
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Kim et al (2021): 116°E, 3.5°N in May 1998

(b)

Westward flux



GW source from convection (Son & Chun 2005, Choi & Chun 2011)

Effects of GW transience on GW momentum-flux intermittency

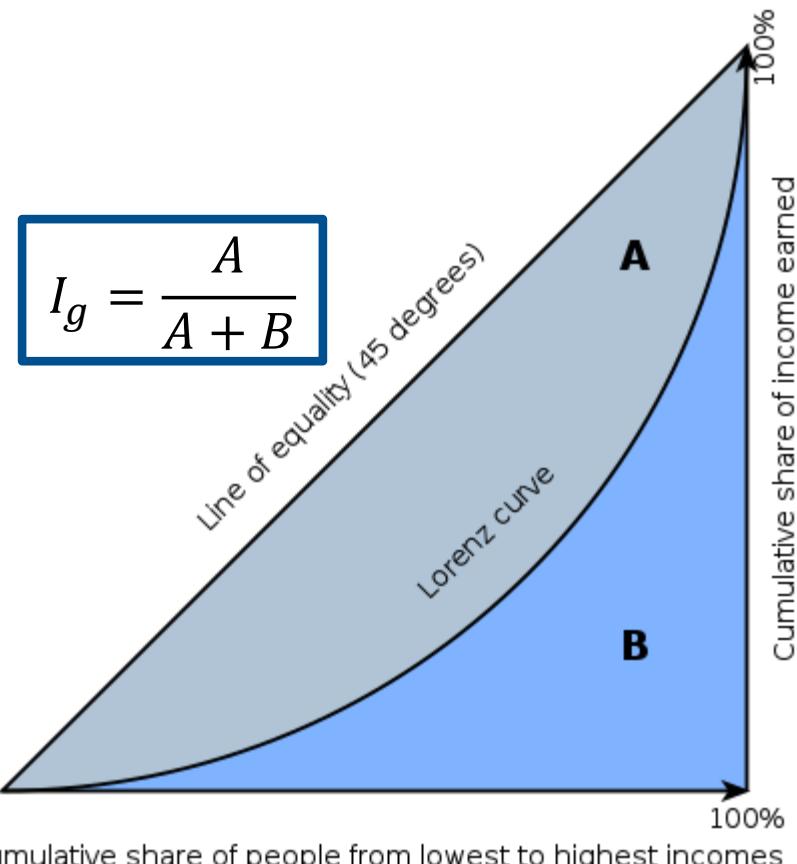


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Gini coefficient: Index for unbalance in distribution



Effects of GW transience on GW momentum-flux intermittency

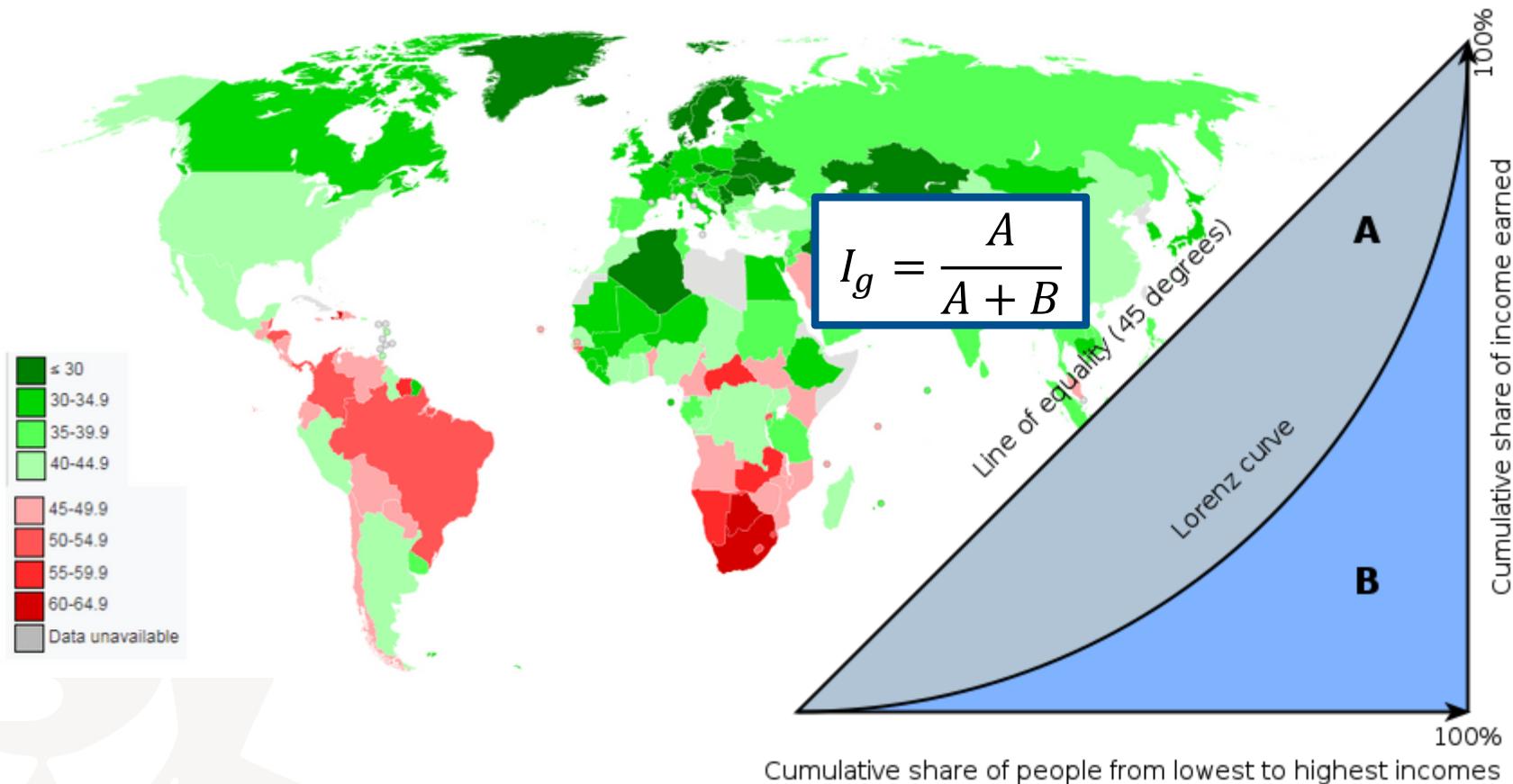


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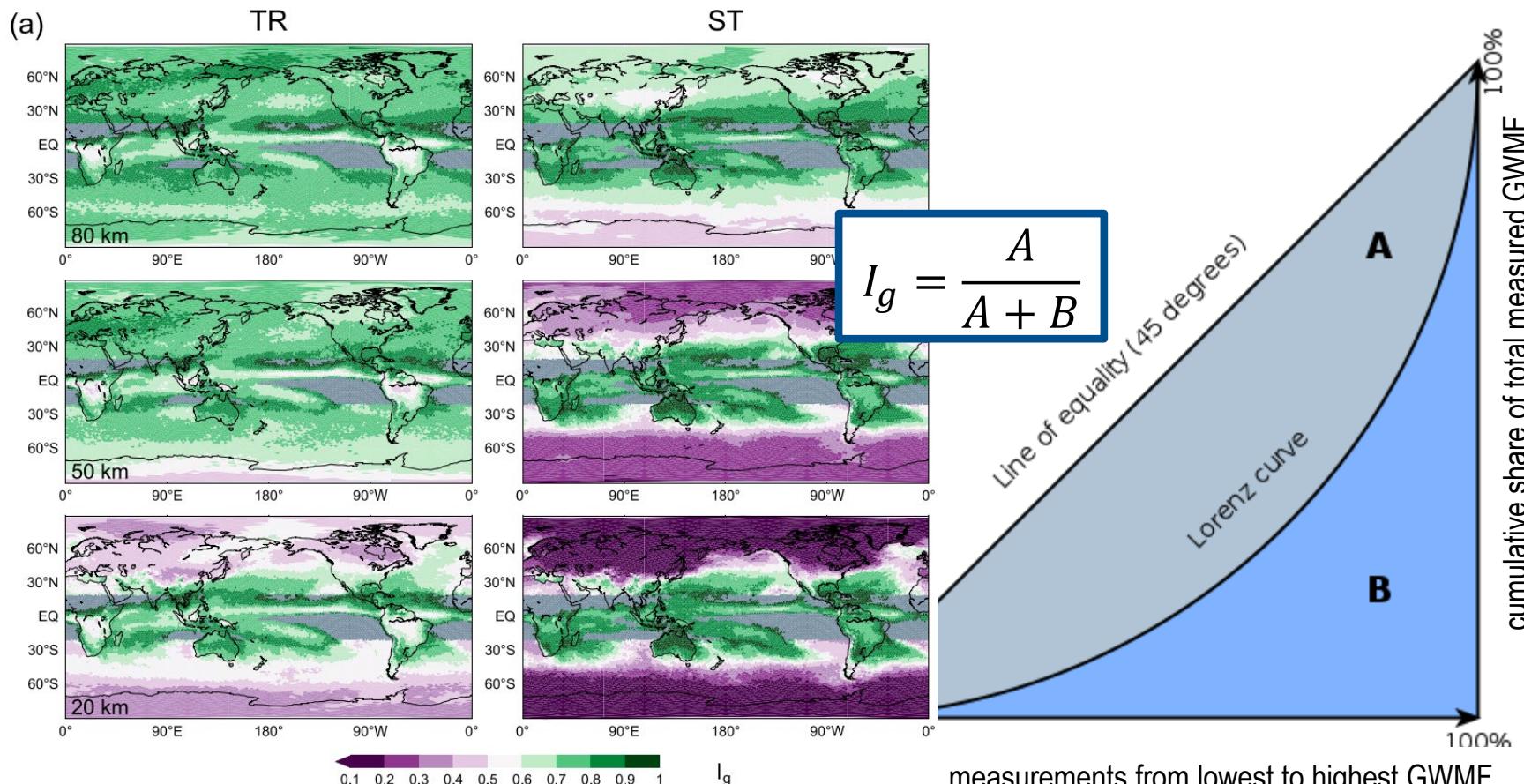
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Gini coefficient: Index for unbalance in distribution



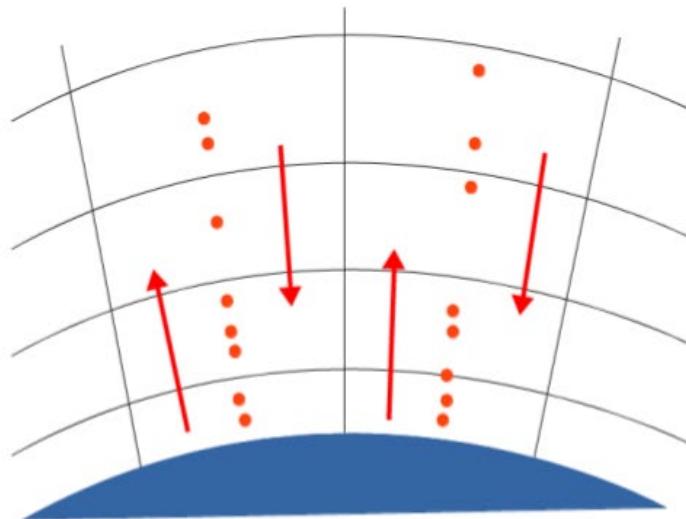
Effects of GW transience on GW momentum-flux intermittency

Kim et al (2021): Gini coefficient for GWMF

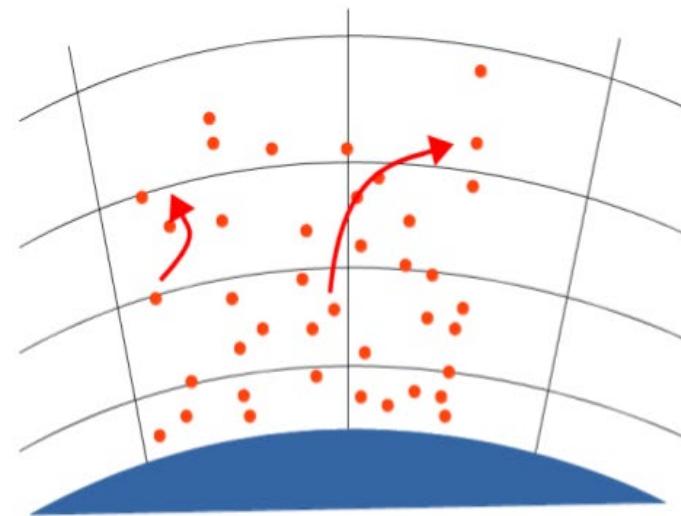


Effects of Horizontal Propagation

1D framework



3D framework



Effects of horizontal propagation: Wave-action budget



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Spatial wave-action density $\mathcal{A} = \int d^3k \mathcal{N}$ satisfies

$$\partial_t \mathcal{A} = -\nabla_h \cdot (\mathbf{c}_{gh} \mathcal{A}) - \partial_z (c_{gz} \mathcal{A}) + D \quad \text{with e.g. } \mathbf{c}_{gh} \mathcal{A} = \int d^3k \mathbf{c}_{gh} \mathcal{N}$$

Time mean

$$0 \approx \frac{\Delta \mathcal{A}}{\Delta t} = -\nabla_h \cdot \langle \mathbf{c}_{gh} \mathcal{A} \rangle - \partial_z \langle c_{gz} \mathcal{A} \rangle + \langle D \rangle$$

Effects of horizontal propagation: Wave-action budget



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Spatial wave-action density $\mathcal{A} = \int d^3k \mathcal{N}$ satisfies

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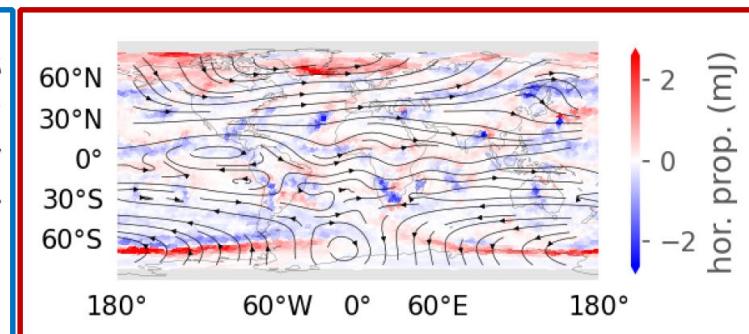
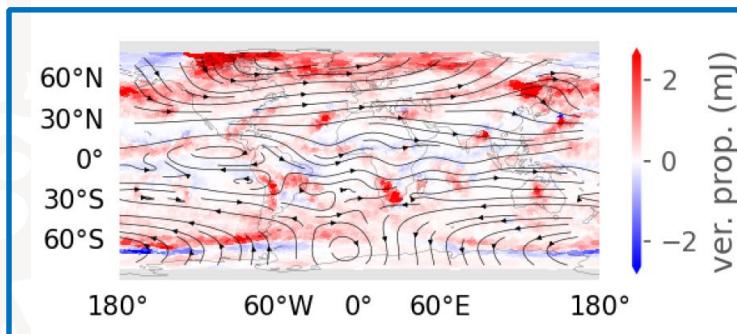
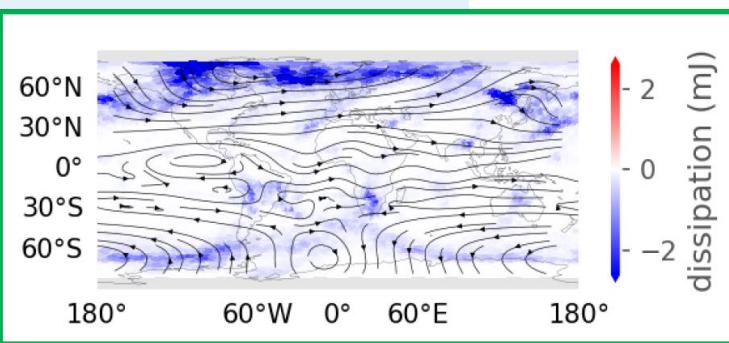
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June

$z \approx 40\text{km}$

(Völker et al 2023, in prep.)



Effects of horizontal propagation: Horizontal distribution GW mom.flux



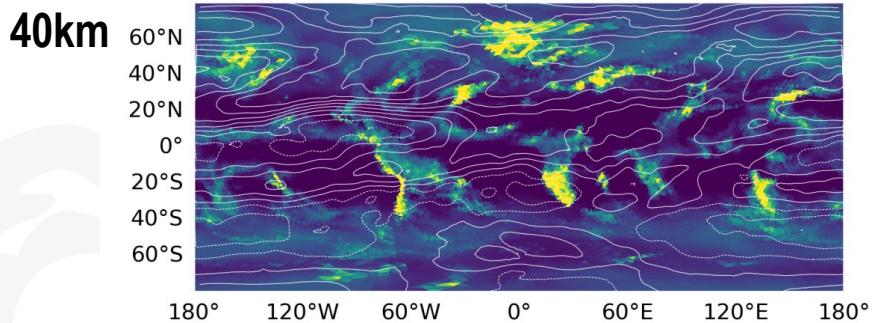
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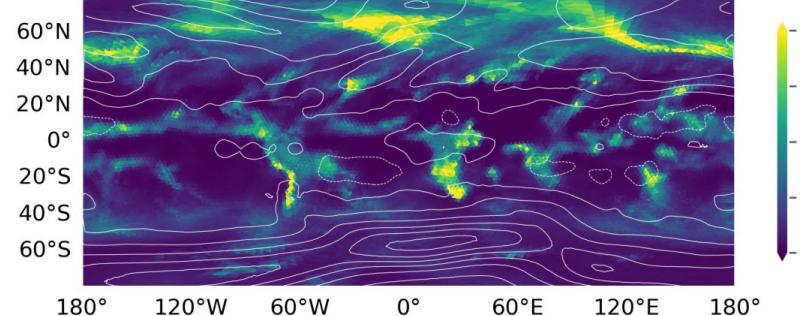
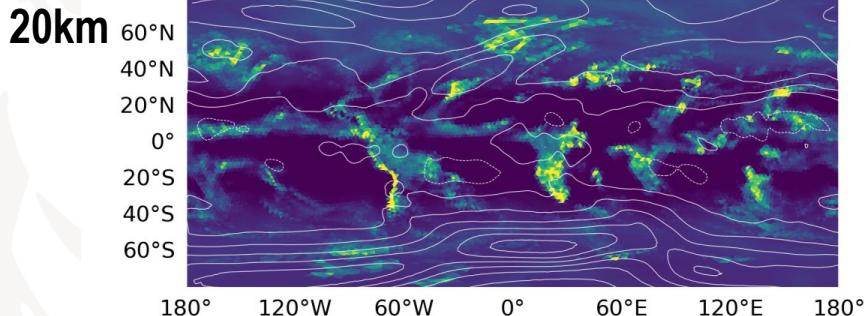
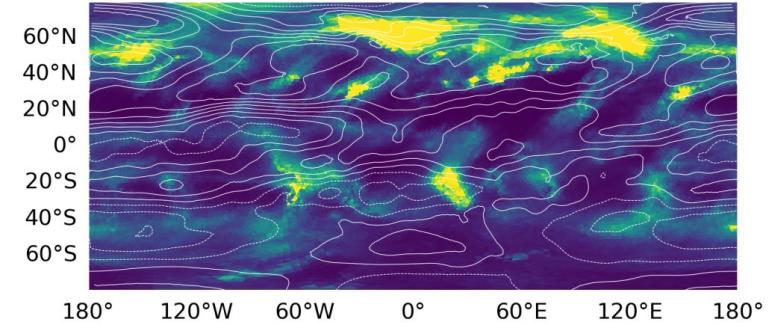
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GW momentum flux November (snapshot) at two altitudes
(Völker et al 2023, in prep.)

1D



3D



Effects of horizontal propagation: GW mom.flux & mean winds



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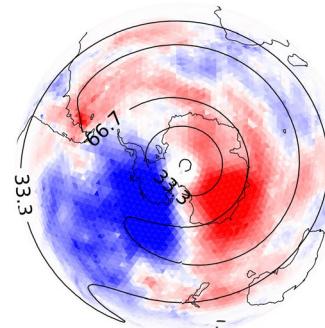
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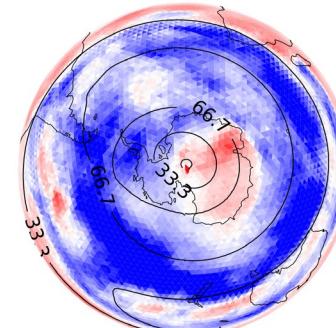
June 1994 Southern Hemisphere **GW meridional momentum flux & mean zonal wind**
(Völker et al 2023, in prep.)

$$\bar{\rho} \langle \overline{w'v'} \rangle$$

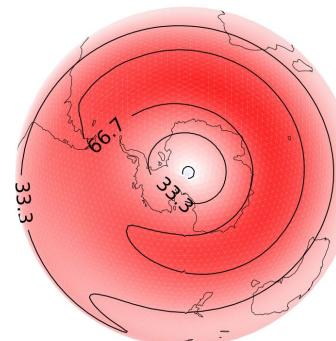
a) MS-GWaM 1D



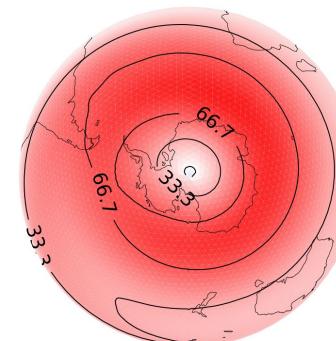
b) MS-GWaM 3D ($z = 39.80$ km)



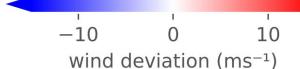
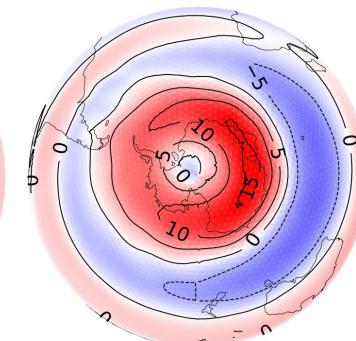
a) MS-GWaM 1D



b) MS-GWaM 3D



c) difference ($z = 39.80$ km)



Effects of horizontal propagation: GWMF intermittency



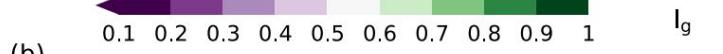
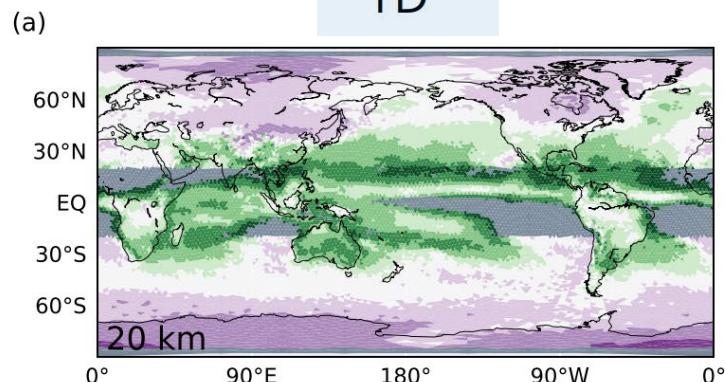
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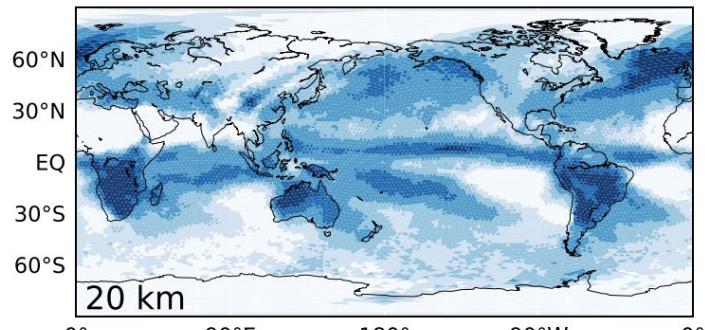
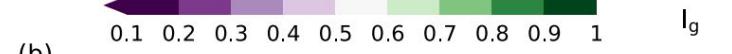
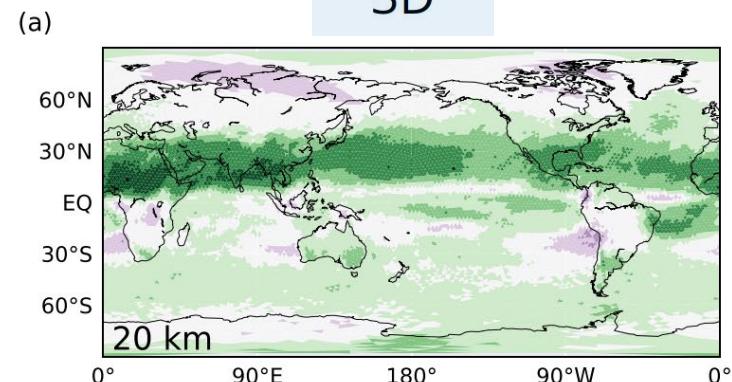
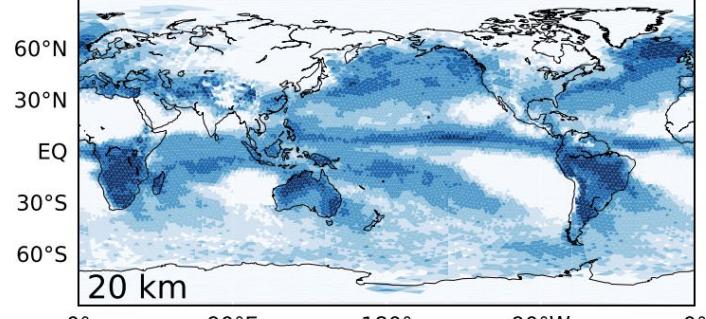
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Kim et al (2023, in prep.)

Gini



GWMF



Effects of horizontal propagation: Zonal-mean zonal wind

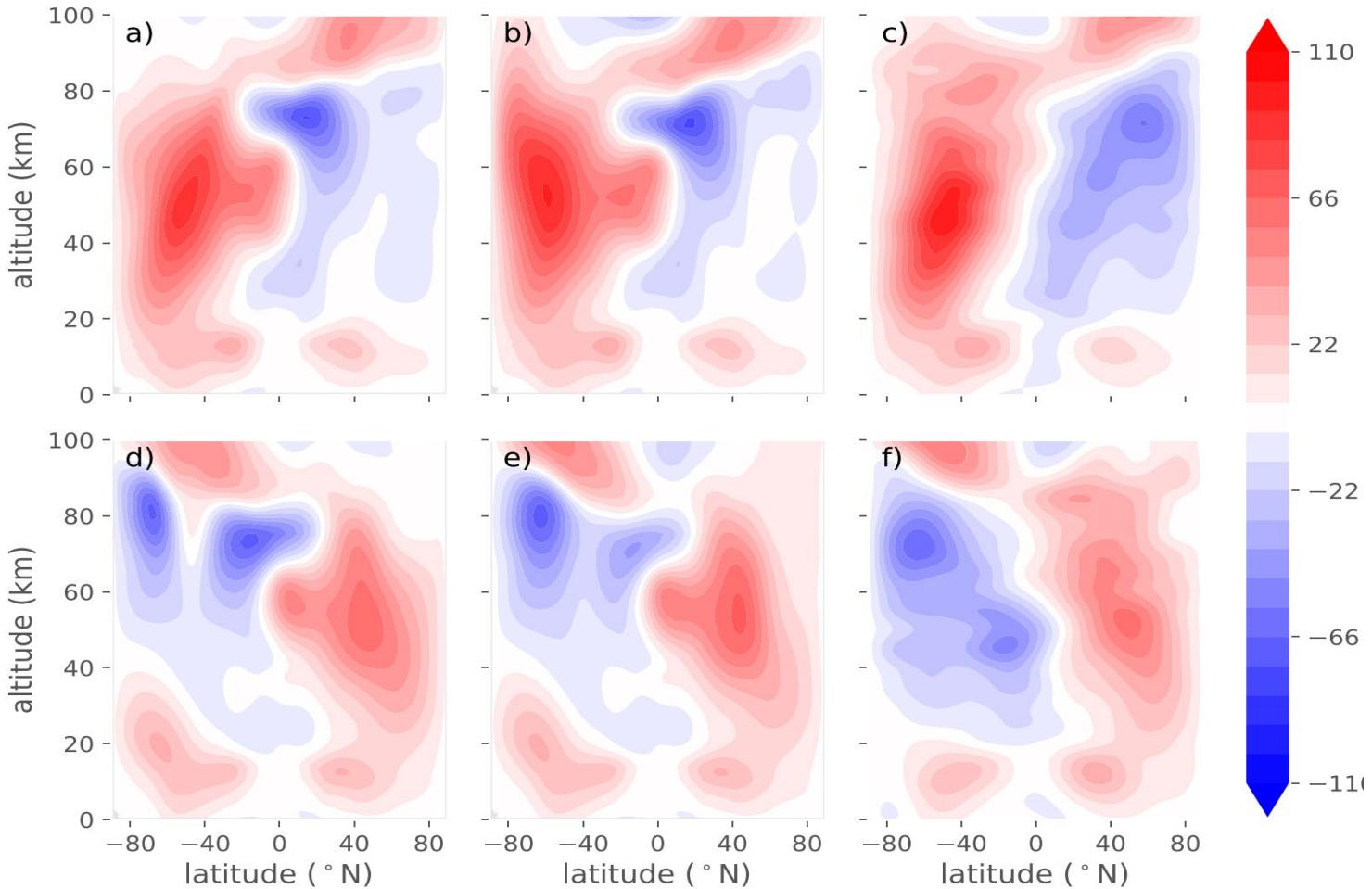


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MS-GWaM 1D

MS-GWaM 3D

Obs (HWM2014)

Effects of horizontal propagation: QBO

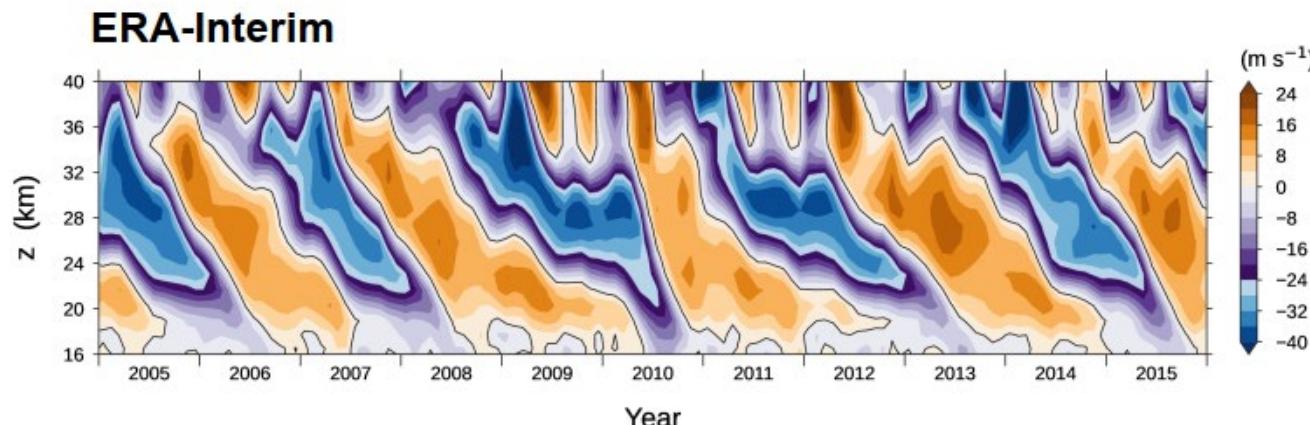


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Quasi-Biennial Oscillation: zonal-mean zonal wind $5^{\circ}\text{S} - 5^{\circ}\text{N}$
(Kim et al 2023, in prep.)



Effects of horizontal propagation: QBO

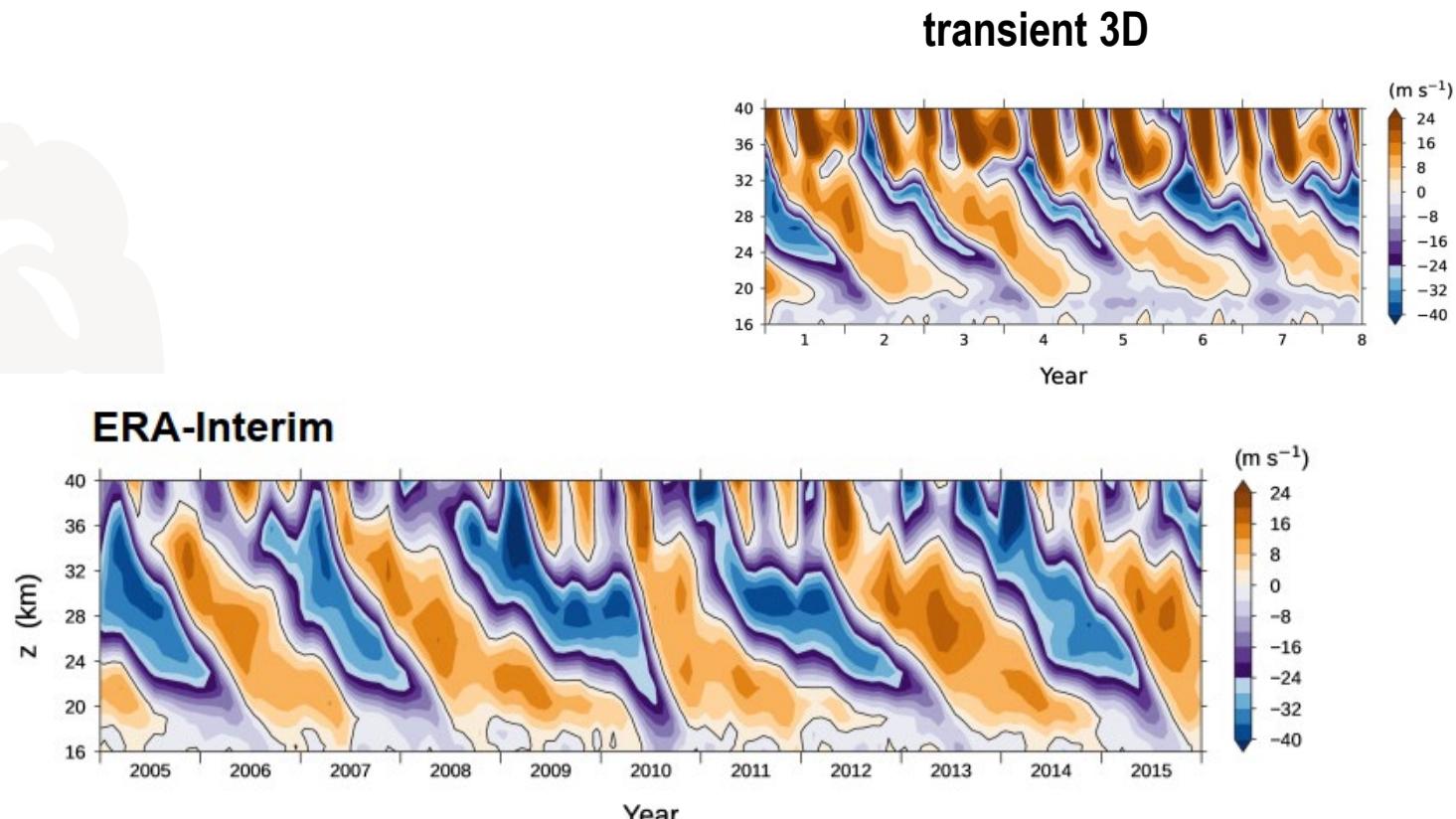


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Quasi-Biennial Oscillation: zonal-mean zonal wind $5^{\circ}\text{S} - 5^{\circ}\text{N}$
(Kim et al 2023, in prep.)



Effects of horizontal propagation: QBO



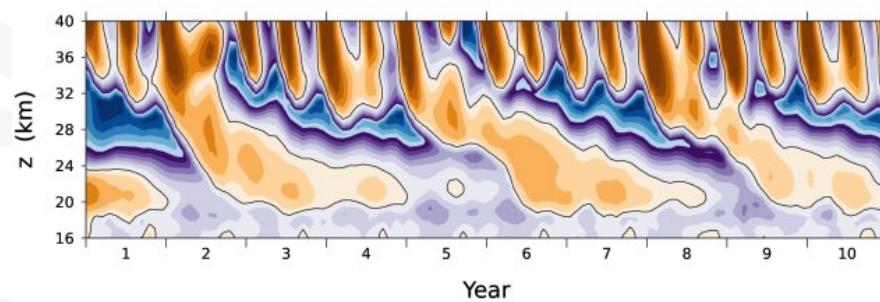
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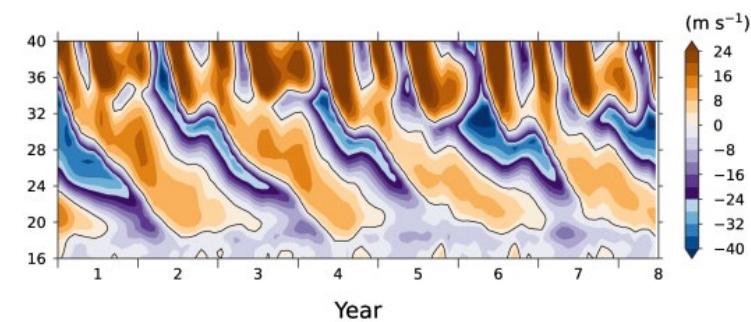
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Quasi-Biennial Oscillation: zonal-mean zonal wind $5^{\circ}\text{S} - 5^{\circ}\text{N}$
(Kim et al 2023, in prep.)

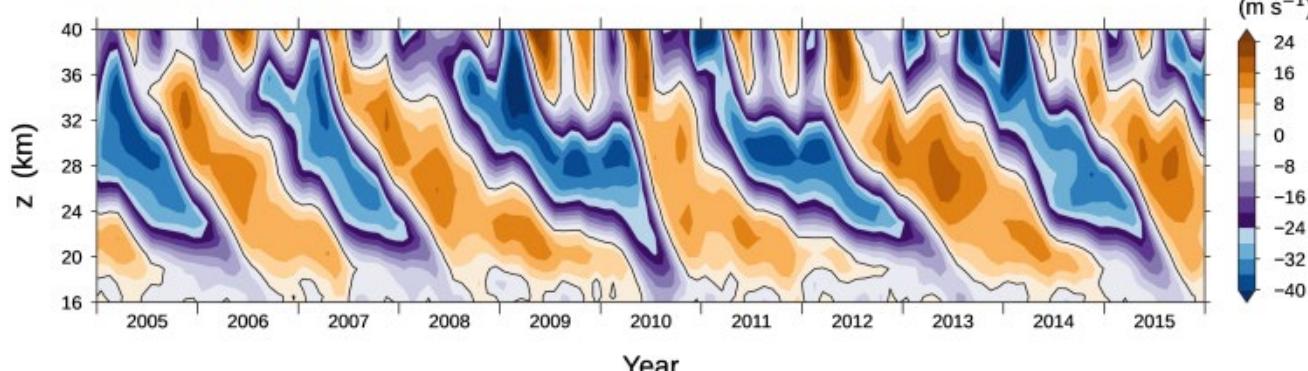
Steady state 1D



transient 3D



ERA-Interim



Effects of horizontal propagation: QBO

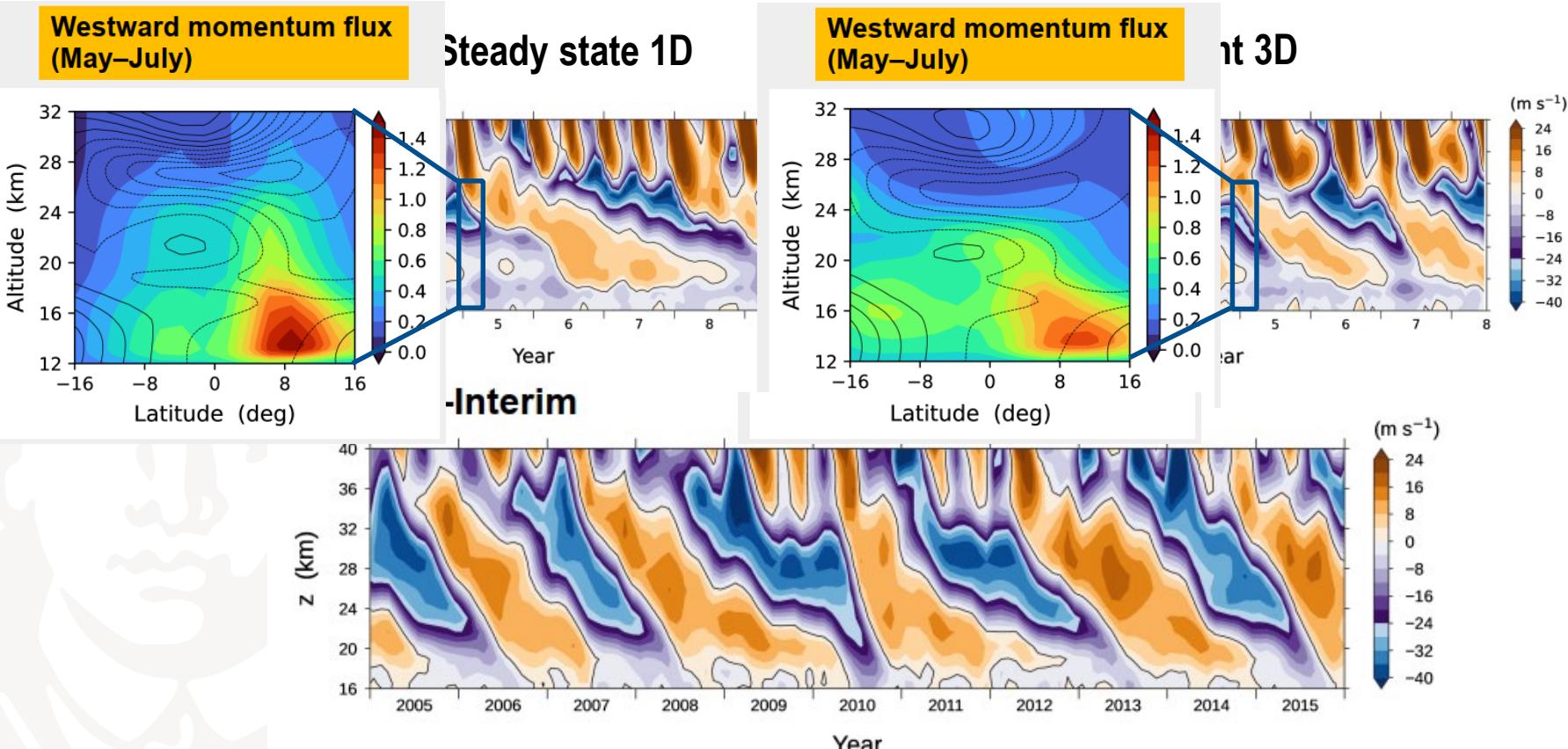


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Quasi-Biennial Oscillation: zonal-mean zonal wind 5°S – 5°N
(Kim et al 2023, in prep.)





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Is it affordable?



MS-GWaM in UA-ICON computational effort



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	UA-ICON / MS-GWaM (3D)
$\Delta x/\text{km}$	160
$\Delta z/\text{km}$	0.02 (BL) – 0.7 (ST) – 4.5 (top)
relative increase comp. time (compared to ICON with classic GWP - Orr et al 2010)	10-40

UA-ICON / MS-GWaM is

- **more expensive** than UA-ICON with **classic GW parameterization**, but

MS-GWaM in UA-ICON computational effort



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	UA-ICON / MS-GWaM (3D)	UA-ICON GW resolving
$\Delta x/\text{km}$	160	5
$\Delta z/\text{km}$	0.02 (BL) – 0.7 (ST) – 4.5 (top)	0.02 (BL) – 0.2 (ST) – 0.2 (top)
relative increase comp. time (compared to ICON with classic GWP - Orr et al 2010)	10-40	240000

UA-ICON / MS-GWaM is

- **more expensive** than UA-ICON with **classic GW parameterization**, but
- **considerably cheaper** than **a wave-resolving set-up of UA-ICON**

Summary & Discussion



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- **Subgrid-scale processes** still present major challenges to the **reliability of climate simulations**
- **Gravity waves (GW)** are a corresponding phenomenon
- Both **transient dynamics** and **horizontal propagation** not represented in present-day parameterizations
- **MS-GWaM in UA-ICON** is 1st prognostic GW model to simulate these effects
- Corresponding **differences are leading order**, e.g.
 - Intermittency
 - Horizontal distributions
 - QBO
- MS-GWaM is more expensive than classic GW parameterizations but much **cheaper than wave-resolving simulations**
- Even when we will be able to resolve all processes even in climate simulations, **conceptional models with a solid theoretical basis** will remain essential for our gain of understanding

Literature



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Book:

Achatz, U., 2022: Atmospheric Dynamics. Springer, Berlin



Journal Papers:

Achatz, U., Ribstein, B., Senf, F., and R. Klein, 2017: The interaction between synoptic-scale balanced flow and a finite-amplitude mesoscale wave field throughout all atmospheric layers: Weak and moderately strong stratification. *Quart. J. R. Met. Soc.*, **143**, 342–361

Bölöni, G., Ribstein, S., Muraschko, J., Sgoff, C., Wei, J., and U. Achatz, 2016: The interaction between atmospheric gravity waves and large-scale flows: an efficient description beyond the non-acceleration paradigm. *J. Atmos. Sci.*, **73**, 4833–4852

Bölöni, G., Kim, Y.-H., Borchert, S., and U. Achatz, 2021: Toward transient subgrid-scale gravity wave representation in atmospheric models. Part I: Propagation model including nondissipative wave–mean-flow interactions. *J. Atmos. Sci.* **78**, 1317–1338

Kim, Y.-H., G. Bölöni, S. Borchert, H.-Y. Chun, and U. Achatz, 2021: Toward transient subgrid-scale gravity wave representation in atmospheric models. Part II: Wave intermittency simulated with convective sources. *J. Atmos. Sci.*, **78**, 1339–1357

Muraschko J, Fruman M, Achatz U, Hickel S, Toledo Y. 2015: On the application of WKB theory for the simulation of the weakly nonlinear dynamics of gravity waves. *Quart. J. R. Met. Soc.* **141**, 676–697.

Wei, J., Bölöni, G., and U. Achatz 2019: Efficient modeling of the interaction of mesoscale gravity waves with unbalanced large-scale flows: Pseudomomentum-flux convergence versus direct approach, *J. Atmos. Sci.*, **76**, 2715–2738

Wilhelm, J., Akylas, T.A., Bölöni, G., Wei, J., Ribstein, B., Klein, R., and U. Achatz, 2018: The interaction between meso- and sub-mesoscale gravity waves. *J. Atmos. Sci.*, **75**, 2257–2280

MS-GWaves: <https://ms-gwaves.iau.uni-frankfurt.de/>