Exercise 2 - Solutions

1. (a) $(DC)_{16} = (1101\ 1100)_2$

$$= 1\cdot 2^7 + 1\cdot 2^6 + 0\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$$

$$= 128 + 64 + 16 + 8 + 4$$

$$= (220)_{10}$$

(b) $(110101)_2 = (0011\ 0101)_2 = (35)_{16}$

$$= 32 + 16 + 4 + 1 = (53)_{10}$$

(c) $k_0 = (1000)_{10} = \underbrace{500}_{k_1}\cdot 2 + \underbrace{0}_{d_0}$; $k_1 = \underbrace{250}_{k_2}\cdot 2 + \underbrace{0}_{d_1}$; $k_2 = \underbrace{125}_{k_3}\cdot 2 + \underbrace{0}_{d_2}$; $k_3 = \underbrace{62}_{k_4}\cdot 2 + \underbrace{1}_{d_3}$

$k_4 = \underbrace{31}_{k_5}\cdot 2 + \underbrace{0}_{d_4}$; $k_5 = \underbrace{15}_{k_6}\cdot 2 + \underbrace{1}_{d_5}$; $k_6 = \underbrace{7}_{k_7}\cdot 2 + \underbrace{1}_{d_6}$; $k_7 = \underbrace{3}_{k_8}\cdot 2 + \underbrace{1}_{d_7}$; $k_8 = \underbrace{1}_{k_9 = d_9}\cdot 2 + \underbrace{1}_{d_8}$

$$\Rightarrow (1000)_{10} = (00\overset{d_9}{1}\underset{d_8}{1}\ \overset{d_7}{1}\underset{d_6}{1}\overset{d_5}{1}\underset{d_4}{0}\ \overset{d_3}{1}\underset{d_2}{0}\overset{d_1}{0}\underset{d_0}{0})_2 = (3E8)_{16}$$

2. (a) $(9)_{10} = (1001)_2$

$(0.5)_{10} = 1 \cdot 2^{-1} = (0.1)_2$

$\Big\}$ $(9.5)_{10} = (1001.1)_2$

(b) $\frac{44}{7} = \frac{42}{7} + \frac{2}{7} = 6 + \frac{2}{7}$ $\qquad (6)_{10} = (110)_2$

To convert fractional part into binary, set $r_0 = \frac{2}{7}$

$k_1 = 2 \cdot r_0 = \frac{4}{7} < 1 \Rightarrow d_{-1} = 0 \qquad r_1 = \frac{4}{7}$

$k_2 = 2 \cdot r_1 = \frac{8}{7} = 1 + \frac{1}{7} \Rightarrow d_{-2} = 1 \qquad r_2 = \frac{1}{7}$

$k_3 = 2 \cdot r_2 = \frac{2}{7} < 1 \Rightarrow d_{-3} = 0 \qquad r_3 = \frac{2}{7}$

sequence repeats!

$\Rightarrow \left(\frac{2}{7}\right)_{10} = 0.\overline{010}$

Altogether: $\left(\frac{44}{7}\right)_{10} = (110.\overline{010})_2$

3. (a) $(1101.0111)_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}$

$= 13 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 13\frac{7}{16} = (13.4375)_{10}$

(b) $(0.0\overline{1001})_2 = 9 \cdot 2^{-5}\,(1 + 2^{-4} + 2^{-8} + \dots)$

$= 9 \cdot 2^{-5} \frac{1}{1 - 2^{-4}} = \frac{9}{32} \frac{1}{\frac{15}{16}} = \frac{3}{2}\frac{1}{5} = (0.3)_{10}$

4. (a) $39 = 2 \cdot 16 + 7 \Rightarrow (39)_{10} = (27)_{16}$

To convert fractional part, set $r_0 = 0.75$

$k_1 = 16 \cdot r_0 = 16\frac{3}{4} = 12 \Rightarrow d_{-1} = C \qquad r_1 = k_1 - d_{-1}$

$\Big\}$ $(39.75)_{10} = (27.C)_{16}$

(b) $(F)_{16} = (15)_{10}$; $(0.42)_{16} = 4 \cdot 16^{-1} + 2 \cdot 16^{-2} = \frac{33}{128} = 0.2578125$

$\Rightarrow (F.42)_{16} = (15.2578125)_{10}$

5.

$$
\begin{aligned}
67 &= 33 \cdot 2 + 1_{d_0} \\
33 &= 16 \cdot 2 + 1_{d_1} \\
16 &= 8 \cdot 2 + 0_{d_2} \\
8 &= 4 \cdot 2 + 0_{d_3} \\
4 &= 2 \cdot 2 + 0_{d_4} \\
2 &= \underset{d_6}{1} \cdot 2 + 0_{d_5}
\end{aligned}
\quad \Bigg\} \quad (67)_{10} = (01000011)_2
$$

To compute two's complement, flip all bits and add 1:

$$
\begin{array}{r}
10111100 \\
+ \quad 1 \\
\hline
10111101
\end{array}
$$

$\Rightarrow (-67)_{10}$ is represented by 10111101 in 8-bit two's complement representation.

6. The number is negative, so need to compute two's complement to find its absolute value:

$$
\begin{array}{r}
01000101 \\
+ \quad 1 \\
\hline
01000110
\end{array}
$$

Now: $(01000110)_2 = 64 + 4 + 2 = 70$

$\Rightarrow$ The number represented by bit pattern 10111010 in 8-bit two's complement representation is $(-70)_{10}$

7. Let i denote the position of the first 1, from the back, of the given bit pattern. In other words, the pattern has the form

$$
\underset{\text{pos } n-1}{d_{n-1}}\ d_{n-2} \cdots d_{i+1}\ \underset{\text{pos } i}{1}\ \underset{\text{pos } i-1}{0} \cdots \underset{\text{pos } 0}{0}
$$

(n bits in total)

Take two's complement:

$$
\begin{array}{r}
d'_{n-1}\ d'_{n-2} \cdots d'_{i+1}\ 0\ 1 \cdots 1 \\
+ \quad 1 \\
\hline
d'_{n-1}\ d'_{n-2} \cdots d'_{i+1}\ 1\ 0 \cdots 0
\end{array}
$$

$\Rightarrow$ bits $0, \ldots, i$ remain the same, bits $i+1, \ldots, n-1$ are flipped

From here, it is obvious that if we do the same thing again, we recover the first pattern.