Exercise 1 - Solutions

1. (a) $$a \underset{\text{identity}}{=} a \wedge 1 \underset{\text{complement}}{=} a \wedge (a \vee a') \underset{\text{distributivity}}{=} (a \wedge a) \vee (a \wedge a') \underset{\text{complement}}{=} (a \wedge a) \vee 0 \underset{\text{identity}}{=} a \wedge a$$

(b) $$a \wedge 0 \underset{\text{complement}}{=} a \wedge (a \wedge a') \underset{\text{associativity}}{=} (a \wedge a) \wedge a' \underset{\text{Dual Th.1}}{=} a \wedge a' \underset{\text{complement}}{=} 0$$

(c) $$a \wedge (a \vee b) \underset{\text{identity}}{=} (a \vee 0) \wedge (a \vee b) \underset{\text{distributivity}}{=} a \vee (0 \wedge b) \underset{\text{Dual Th.2}}{=} a \vee 0 \underset{\text{identity}}{=} a$$

(d) We need to check that the RHS, $a' \vee b'$, satisfies the properties of the complement of $a \wedge b$.

- $$(a \wedge b) \vee (a' \vee b') \underset{\text{distributivity}}{=} (a \vee (a' \vee b')) \wedge (b \vee (a' \vee b')) \underset{\substack{\text{associativity}\\ \text{commutativity}\\ \text{complement}}}{=} (1 \vee b') \wedge (1 \vee a') \underset{\text{Th.2}}{=} 1 \wedge 1 \underset{\substack{\text{identity}\\ \text{(or Th.1)}}}{=} 1$$

- $$(a \wedge b) \wedge (a' \vee b') \underset{\text{distributivity}}{=} ((a \wedge b) \wedge a') \vee ((a \wedge b) \wedge b') \underset{\substack{\text{associativity}\\ \text{commutativity}\\ \text{complement}}}{=} (0 \wedge b) \vee (0 \wedge a) \underset{\text{Dual Th.2}}{=} 0 \vee 0 \underset{\substack{\text{identity}\\ \text{(or Dual Th.1)}}}{=} 0$$

2. (a) True: Associative law

(b) False. Counterexample: $a = 0$, $c = 1$. Then

$$\left.\begin{array}{l} \text{LHS} = 0 \wedge (b \vee c) = 0 \\ \text{RHS} = (a \wedge b) \vee 1 = 1 \end{array}\right\} \text{Since these don't match, the statement is false.}$$

(c) True. Distributive law

(d) True. Distributive law (Dual of statement c)

(e) True. De Morgan's law.

3. (a) $$(a \wedge b) \vee (a \wedge b') \underset{\substack{\uparrow \\ \text{distributivity}}}{=} a \wedge (b \vee b') \underset{\substack{\uparrow \\ \text{complement}}}{=} a \wedge 1 \underset{\substack{\uparrow \\ \text{identity}}}{=} a$$

(b) $$(a \wedge b') \vee b \underset{\substack{\uparrow \\ \text{distributivity}}}{=} (a \vee b) \wedge (b' \vee b) \underset{\substack{\uparrow \\ \text{complement}}}{=} (a \vee b) \wedge 1 \underset{\substack{\uparrow \\ \text{identity}}}{=} a \vee b$$

(c) $$(a \vee b) \wedge (b \vee c) \wedge (a' \vee c) = (a \vee b) \wedge \underbrace{(0 \vee b \vee c)} \wedge (a' \vee c) \qquad \text{(identity)}$$

$$= (a \wedge a') \vee b \vee c \qquad \text{(complement)}$$

$$= (a \vee b \vee c) \wedge (a' \vee b \vee c) \qquad \text{(distributivity)}$$

$$= \Big[(a \vee b) \wedge ((a \vee b) \vee c)\Big] \wedge \Big[(a' \vee c) \wedge ((a' \vee c) \vee b)\Big] \qquad \text{(assoc., comm.)}$$

$$= (a \vee b) \wedge (a' \vee c) \qquad \text{(absorption)}$$

4. $$(a' \wedge b' \wedge c) \vee (a' \wedge b \wedge c) \vee (a \wedge b' \wedge c') \vee (a \wedge b \wedge c')$$

$$= \Big[a' \wedge c \wedge \underbrace{(b' \vee b)}_{=1}\Big] \vee \Big[a \wedge c' \wedge \underbrace{(b' \vee b)}_{=1}\Big]$$

$$= (a' \wedge c) \vee (a \wedge c') = a \text{ XOR } c$$

5. (a) $$z = b' \wedge (b \vee a') = \underbrace{(b' \wedge b)}_{=0} \vee (b' \wedge a') = a' \wedge b' = (a \vee b)' = a \text{ NOR } b$$

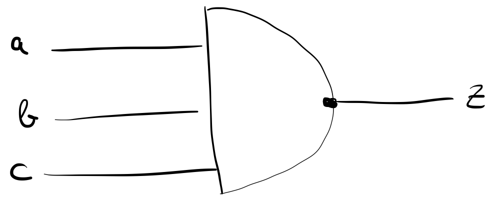
(b) $$z = (a \wedge b' \wedge c) \vee (a' \wedge b) \vee (a \wedge c)'$$

$$= (a \wedge b' \wedge c) \vee \underbrace{(a' \wedge b) \vee a'}_{= a' \text{ by absorption}} \vee c' \qquad \text{(De Morgan}$$

$$= (a \wedge c \wedge b') \vee (a \wedge c)' \qquad \text{(commutativity, De Morgan)}$$

$$= \Big(\underbrace{(a \wedge c) \vee (a \wedge c)'}_{=1 \text{ (complement)}}\Big) \wedge \Big(b' \vee (a \wedge c)'\Big) \underset{\substack{\uparrow \\ \text{identity, De Morgan}}}{=} (a \wedge b \wedge c)'$$

So the circuit can be implemented as a triple NAND-gate:



Implementation in non-simplified form: