

# Foundations of Information Systems

## Makeup Exam

April 7, 2026

1. Are the following Boolean algebra expressions true or false? If true, provide a proof. If false, provide a counterexample.

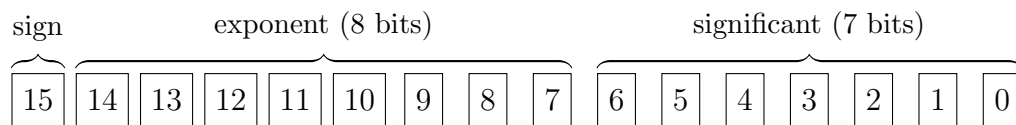
(a)  $a' \vee b' = (a \vee b)'$

(b)  $a \vee (a \wedge b) = a$

(c)  $(a \wedge c) \vee (a \wedge b \wedge c) = a \wedge c$

(5+5+5)

2. **bfloat16** is a 16-bit floating point format where the bits are allocated as follows.



Per IEEE 754 convention, the exponent bias is 127.

- (a) Convert the data word

101111111 0100000

from **bfloat16** to decimal.

- (b) What is the maximal relative error when converting an arbitrary decimal to **bfloat16**?

(You may state your answer as a fraction or in terms of a power of 2; no need to convert to decimal.)

- (c) What is special about **bfloat16** and why is it getting popular in neural network architectures?

(5+5+5)

3. Draw a deterministic (!) finite state machine that accepts input strings if and only if they contain the substring 1010. (10)

4. (a) In a RAID-5 array, one disk out of 5 has failed. The others contain the bit sequences 11111111..., 01101000..., 00000000..., 11100001.... Reconstruct the beginning of the bit sequence of the failed disk.

- (b) The RAID-5 array consists of disks with a capacity of 1 TB each and a rate of *unrecoverable read errors* (URE), defined as the the probability of a single bit error during read, of  $URE = 10^{-14}$ . Assuming that single-bit errors are statistically independent, estimate the probability that the array can be fully recovered.

(Simplify your answer as much as reasonably possible, but you do not need to evaluate to a number; take  $1 \text{ TB} = 2^{40} \text{ B.}$ )

- (c) Assume that the hard drives have a native block size of  $4 \text{ kB} = 2^{12} \text{ B.}$  A block can only be read fully, or not at all. What is the rate of unrecoverable block reads? Use this number to explain why adding another disk and running the array in RAID-6 mode makes the recovery process much safer.

(5+5+5)

5. Encode the data word

1001 0111 010

in Hamming-(16,11) encoding.

Use the bit-ordering convention employed in class. (5)

6. Give a simple example of a network with distance vector routing that illustrates the saying “good news travels fast, bad news travels slow”.

Your answer should provide details of a “good news” and a “bad news” event. (10)

7. You are given the following simplified library database schema:

```
Book(BookID, Title, Author)
Patron(PatronID, Name, Address)
Loan(DueDate, PatronID, BookID)
```

- (a) Underline the primary keys and dashed-underline all foreign keys.
- (b) Write the following queries either in relational algebra or in SQL.
- Find the names of all patrons who have borrowed at least one book.
  - Find the names of all authors that patron Jane Smith is currently reading.

(3+3+4)