Exercise 3 - Solutions

1. (a) $(9)_{10} = (1001)_2$

$(0.5)_{10} = 1 \cdot 2^{-1} = (0.1)_2$

$\Big\} \quad (9.5)_{10} = (1001.1)_2$

(b) $\frac{44}{7} = \frac{42}{7} + \frac{2}{7} = 6 + \frac{2}{7}$ $\qquad (6)_{10} = (110)_2$

To convert fractional part into binary, set $x_0 = \frac{2}{7}$

$x_1 = 2 \cdot x_0 = \frac{4}{7} < 1 \quad \Rightarrow \quad d_{-1} = 0 \qquad r_1 = \frac{4}{7}$

$x_2 = 2 \cdot r_1 = \frac{8}{7} = 1 + \frac{1}{7} \quad \Rightarrow \quad d_{-2} = 1 \qquad r_2 = \frac{1}{7}$

$x_3 = 2 \cdot r_2 = \frac{2}{7} < 1 \quad \Rightarrow \quad d_{-3} = 0 \qquad r_3 = \frac{2}{7}$

sequence repeats!

$\Rightarrow \left(\frac{2}{7}\right)_{10} = 0.\overline{010}$

Altogether: $\left(\frac{44}{7}\right)_{10} = (110.\overline{010})_2$

2. (a) $(1101.0111)_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}$

$= 13 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 13\frac{7}{16} = (13.4375)_{10}$

(b) $(0.0\overline{1001})_2 = 9 \cdot 2^{-5} (1 + 2^{-4} + 2^{-8} + \dots)$

$= 9 \cdot 2^{-5} \frac{1}{1 - 2^{-4}} = \frac{9}{32} \frac{1}{\frac{15}{16}} = \frac{3}{2} \frac{1}{5} = (0.3)_{10}$

3. (a) $39 = 2 \cdot 16 + 7 \quad \Rightarrow \quad (39)_{10} = (27)_{16}$

To convert fractional part, set $x_0 = 0.75$

$x_1 = 16 \cdot x_0 = 16 \frac{3}{4} = 12 \quad \Rightarrow \quad d_{-1} = C \qquad r = 0$

$\Big\} \quad (39.75)_{10} = (27.C)_{16}$

(b) $(F)_{16} = (15)_{10}$; $(0.42)_{16} = 4 \cdot 16^{-1} + 2 \cdot 16^{-2} = \frac{33}{128} = 0.2578125$

$\Rightarrow (F.42)_{16} = (15.2578125)_{10}$

4.

$$\left.\begin{aligned}
67 &= 33\cdot 2 + 1_{d_0}\\
33 &= 16\cdot 2 + 1_{d_1}\\
16 &= 8\cdot 2 + 0_{d_2}\\
8 &= 4\cdot 2 + 0_{d_3}\\
4 &= 2\cdot 2 + 0_{d_4}\\
2 &= \underset{d_6}{1}\cdot 2 + 0_{d_5}
\end{aligned}\right\} \quad (67)_{10} = (01000011)_2$$

To compute two's complement, flip all bits and add 1:

$$\begin{array}{r} 10111100 \\ +\quad 1 \\ \hline 10111101 \end{array}$$

$\Rightarrow (-67)_{10}$ is represented by 10111101 in 8-bit two's complement representation.

5. The number is negative, so need to compute two's complement to find its absolute value:

$$\begin{array}{r} 0100\,0101 \\ +\quad 1 \\ \hline 01000110 \end{array}$$

Now: $(01000110)_2 = 64 + 4 + 2 = 70$

$\Rightarrow$ The number represented by bit pattern 10111010 in 8-bit two's complement representation is $(-70)_{10}$

6. Let i denote the position of the first 1, from the back, of the given bit pattern. In other words, the pattern has the form

$$\underset{\text{pos } n-1}{d_{n-1}}\ d_{n-2}\ \cdots\ d_{i+1}\ \underset{\text{pos } i}{1}\ \underset{\text{pos } i-1}{0}\ \cdots\ \underset{\text{pos } 0}{0} \qquad (n \text{ bits in total})$$

Take two's complement:

$$\begin{array}{r} d'_{n-1}\ d'_{n-2}\ \cdots\ d'_{i+1}\ 0\ 1\ \cdots\ 1 \\ +\qquad\qquad\qquad 1 \\ \hline d'_{n-1}\ d'_{n-2}\ \cdots\ d'_{i+1}\ 1\ 0\ \cdots\ 0 \end{array}$$

$\Rightarrow$ bits $0, \ldots, i$ remain the same, bits $i+1, \ldots, n-1$ are flipped

From here, it is obvious that if we do the same thing again, we recover the first pattern.