Exercise 2 - Solutions

1. (a) $(a \wedge b) \vee (a \wedge b') \underset{\text{distributivity}}{=} a \wedge (b \vee b') \underset{\text{complement}}{=} a \wedge 1 \underset{\text{identity}}{=} a$

(b) $(a \wedge b') \vee b \underset{\text{distributivity}}{=} (a \vee b) \wedge (b' \vee b) \underset{\text{complement}}{=} (a \vee b) \wedge 1 \underset{\text{identity}}{=} a \vee b$

(c) $(a \vee b) \wedge (b \vee c) \wedge (a' \vee c) = (a \vee b) \wedge \underbrace{(0 \vee b \vee c)} \wedge (a' \vee c)$ (identity)

$= (a \wedge a') \vee b \vee c$ (complement)

$= (a \vee b \vee c) \wedge (a' \vee b \vee c)$ (distributivity)

$= \left[(a \vee b) \wedge ((a \vee b) \vee c)\right] \wedge \left[(a' \vee c) \wedge ((a' \vee c) \vee b)\right]$ (assoc., comm.)

$= (a \vee b) \wedge (a' \vee c)$ (absorption)

2. $(a' \wedge b' \wedge c) \vee (a' \wedge b \wedge c) \vee (a \wedge b' \wedge c') \vee (a \wedge b \wedge c')$

$= \left[a' \wedge c \wedge \underbrace{(b' \vee b)}_{=1}\right] \vee \left[a \wedge c' \wedge \underbrace{(b' \vee b)}_{=1}\right]$

$= (a' \wedge c) \vee (a \wedge c') = a \text{ XOR } c$

3. (a) $z = b' \wedge (b \vee a') = \underbrace{(b' \wedge b)}_{=0} \vee (b' \wedge a') = a' \wedge b' = (a \vee b)' = a \text{ NOR } b$

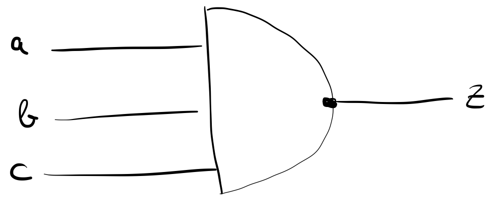
(b) $z = (a \wedge b' \wedge c) \vee (a' \wedge b) \vee (a \wedge c)'$

$= (a \wedge b' \wedge c) \vee \underbrace{(a' \wedge b) \vee a'}_{= a' \text{ by absorption}} \vee c'$ (De Morgan

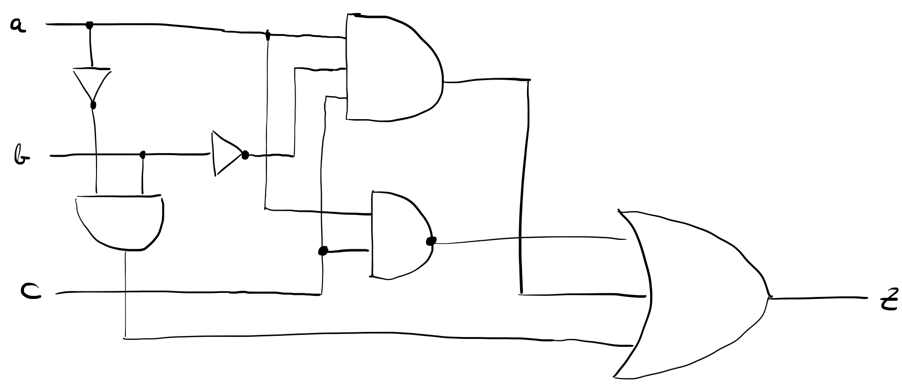
$= (a \wedge c \wedge b') \vee (a \wedge c)'$ (commutativity, De Morgan)

$= (\underbrace{(a \wedge c) \vee (a \wedge c)'}_{=1 \text{ (complement)}}) \wedge (b' \vee (a \wedge c)') \underset{\text{identity, De Morgan}}{=} (a \wedge b \wedge c)'$

So the circuit can be implemented as a triple NAND-gate:



Implementation in non-simplified form:



4. (a) $(DC)_{16} = (1101\ 1100)_2$

$$= 1\cdot 2^7 + 1\cdot 2^6 + 0\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$$

$$= 128 + 64 + 16 + 8 + 4$$

$$= (220)_{10}$$

(b) $(110101)_2 = (0011\ 0101)_2 = (35)_{16}$

$$= 32 + 16 + 4 + 1 = (53)_{10}$$

(c) $k_0 = (1000)_{10} = \underbrace{500}_{k_1}\cdot 2 + \underbrace{0}_{d_0}$; $k_1 = \underbrace{250}_{k_2}\cdot 2 + \underbrace{0}_{d_1}$; $k_2 = \underbrace{125}_{k_3}\cdot 2 + \underbrace{0}_{d_2}$; $k_3 = \underbrace{62}_{k_4}\cdot 2 + \underbrace{1}_{d_3}$

$k_4 = \underbrace{31}_{k_5}\cdot 2 + \underbrace{0}_{d_4}$; $k_5 = \underbrace{15}_{k_6}\cdot 2 + \underbrace{1}_{d_5}$; $k_6 = \underbrace{7}_{k_7}\cdot 2 + \underbrace{1}_{d_6}$; $k_7 = \underbrace{3}_{k_8}\cdot 2 + \underbrace{1}_{d_7}$; $k_8 = \underbrace{1}_{k_9 = d_9}\cdot 2 + \underbrace{1}_{d_8}$

d_9 d_7 d_5 d_3 d_1

$$\Rightarrow (1000)_{10} = (0011\ 1110\ 1000)_2 = (3E8)_{16}$$

d_8 d_6 d_4 d_2 d_0