Exercise 1 - Solutions

1. (a) $$a \underset{\text{identity}}{=} a \wedge 1 \underset{\text{complement}}{=} a \wedge (a \vee a') \underset{\text{distributivity}}{=} (a \wedge a) \vee (a \wedge a') \underset{\text{complement}}{=} (a \wedge a) \vee 0 \underset{\text{identity}}{=} a \wedge a$$

(b) $$a \wedge 0 \underset{\text{complement}}{=} a \wedge (a \wedge a') \underset{\text{associativity}}{=} (a \wedge a) \wedge a' \underset{\text{Dual Th.1}}{=} a \wedge a' \underset{\text{complement}}{=} 0$$

(c) $$a \wedge (a \vee b) \underset{\text{identity}}{=} (a \vee 0) \wedge (a \vee b) \underset{\text{distributivity}}{=} a \vee (0 \wedge b) \underset{\text{Dual Th.2}}{=} a \vee 0 \underset{\text{identity}}{=} a$$

(d) We need to check that the RHS, $a' \vee b'$, satisfies the properties of the complement of $a \wedge b$.

- $$(a \wedge b) \vee (a' \vee b') \underset{\text{distributivity}}{=} (a \vee (a' \vee b')) \wedge (b \vee (a' \vee b')) \underset{\substack{\text{associativity} \\ \text{commutativity} \\ \text{complement}}}{=} (1 \vee b') \wedge (1 \vee a') \underset{\text{Th.2}}{=} 1 \wedge 1 \underset{\substack{\text{identity} \\ \text{(or Th.1)}}}{=} 1$$

- $$(a \wedge b) \wedge (a' \vee b') \underset{\text{distributivity}}{=} ((a \wedge b) \wedge a') \vee ((a \wedge b) \wedge b') \underset{\substack{\text{associativity} \\ \text{commutativity} \\ \text{complement}}}{=} (0 \wedge b) \vee (0 \wedge a) \underset{\text{Dual Th.2}}{=} 0 \vee 0 \underset{\substack{\text{identity} \\ \text{(or Dual Th.1)}}}{=} 0$$

2. (a) True: Associative law

(b) False. Counterexample: $a = 0$, $c = 1$. Then

$$\left.\begin{aligned} \text{LHS} &= 0 \wedge (b \vee c) = 0 \\ \text{RHS} &= (a \wedge b) \vee 1 = 1 \end{aligned}\right\} \text{ Since these don't match, the statement is false.}$$

(c) True. Distributive law

(d) True. Distributive law (Dual of statement c)

(e) True. De Morgan's law.