Foundations of Information Systems

Winter Semester 2024–25, Exercise 4

For discussion on Wednesday, November 13, 2024

1. Identify values of x for which there is a substantial growth of relative error when the formula is evaluated in floating point arithmetic (due to subtraction of almost equal numbers). Then suggest an alternate formula that improves accuracy for the problematic range of x.

   (a) $\dfrac{1-(1-x)^3}{x}$

   (b) $\dfrac{1-\sqrt{1-x^2}}{x}$

   (c) $\dfrac{1-\sec x}{\tan^2 x}$

   Hint: Recall that $\sec x = (\cos x)^{-1}$; use the well-known trigonometric identity $\sec^2 x = \tan^2 x + 1$.

2. Does the distributive law hold for floating point computations?

3. Let $a = 1.0101 \cdot 2^5$ and $b = 1.1101 \cdot 2^3$ be numbers in binary floating point with a 5-bit significant. Compute $a \oplus b$ and $a \ominus b$ by hand and state the absolute and the relative error.

4. Convert the following single-precision floating bit representation to decimal:

   $$1\,10001010\,11000010000000000000000$$

5. Adapt the computation of propagation of floating point error to the case of floating point division. You should find that floating point division has moderate growth of relative error for all numbers.