Foundations of Information Systems

Winter Semester 2023–24, Exercise 1

For discussion on Wednesday, October 25, 2023

1. In class, we have proved a number of theorems consecutively, each proof referring to some of the previous results. We have also learned that each theorem in Boolean algebra has a "dual theorem", obtained by simultaneously exchanging $\wedge \leftrightarrow \vee$ and $0 \leftrightarrow 1$.

   Prove the following sequence of theorems dual to the ones proved in class. Here, the set of elements of the Boolean algebra is B and the statements are to be true for all $a, b \in B$.

   (a) Dual Theorem 1: $a \wedge a = a$

   (b) Dual Theorem 2: $a \wedge 0 = 0$

   (c) Dual "Absorption Theorem": $a \wedge (a \vee b) = a$

   (d) Dual "De Morgan's Law": $(a \wedge b)' = a' \vee b'$

   Note: Write out the proofs from scratch without referring to the dual symmetry principle. If you get stuck, you can always look at the lecture notes: each step you take should be the dual of one of the steps taken in class.

2. Determine whether the following statements are true or false. If true, state the name of the respective Boolean algebra axiom or elementary theorem (a proof is not required). If false, state a counterexample.

   (a) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

   (b) $a \wedge (b \vee c) = (a \wedge b) \vee c$

   (c) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

   (d) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

   (e) $(a \wedge b)' = a' \vee b'$