1. Determine whether the following statements are true or false. If true, state the name of the respective Boolean algebra axiom or elementary theorem (a proof is not required). If false, state a counterexample.

   (a) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

   (b) $a \wedge (b \vee c) = (a \wedge b) \vee c$

   (c) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

   (d) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

   (e) $(a \wedge b)' = a' \vee b'$

(2+2+2+2+2)

(a) True, associative law.

(b) False. Take $a=0$, $b=1$, $c=1$: $a \wedge (b \vee c) = 0$ but $(a \wedge b) \vee c = 1$

(c) True, distributive law.

(d) True, (second) distributive law

(e) True, de Morgan's law.

2. (a) Convert the decimal number 7.1875 to binary.

(b) Using an 8-bit allocation, first convert each of the following integers to the two's complement binary representation, perform the operation, then convert the result back to decimal:

$$(-23) + 19$$

(5+5)

(a) $(7)_{10} = (111)_2$

$$0.1875 = 0 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} + \underbrace{0.0625}_{= 1 \cdot 2^{-4} + 0}$$

$\Rightarrow$ $(7.1875)_{10} = (111.0011)_2$

(b) $(23)_{10} = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = (10111)_2$

$(19)_{10} = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = (10011)_2$

- 8-bit two's complement of $(10111)_2 = (0001\ 0111)_2$:

$$\begin{array}{r} 1110\ 1000 \\ +\qquad\quad 1 \\ \hline 11101001 \end{array}$$

- Computation in binary:

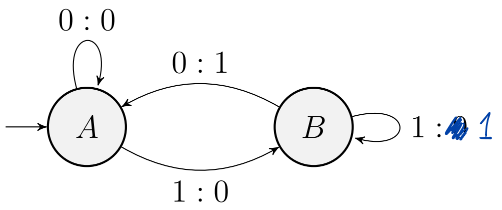
$$\begin{array}{r} 11101001 \\ 00010011 \\ \hline 11111100 \end{array}$$

The sign bit is set, so this is a negative number. Need to compute two's complement to interpret:

$$\begin{array}{r} 00000011 \\ +\qquad 1 \\ \hline 00000100 \end{array} = (4)_{10}$$

Thus, the result of the computation is $(-4)_{10}$, as expected.

3. Consider the following finite state transducer:



(a) Describe the function that is performed by this transducer.

(b) The transducer can be used as a "multiply-by-two" machine. How do you need to organize input and output of the transducer so that it performs this function correctly?

(5+5)

(a) The transducer reads in a binary sequence digit-by-digit.

The output sequence starts with 0, then each output digit is the previous input digit.

(b) Binary multiplication by 2 is a left-shift padded with 0.

Thus, the transducer can perform multiplication by 2 by

- reading the binary number right-to-left
- reading a dummy digit to have the left-most digit come out
- interpreting the output right-to-left.

4. (a) State the four necessary conditions for deadlock.

(b) Describe the difference between deadlock and starvation.

(5+5)

(a)
- mutual exclusion
- holding while waiting
- no pre-emption
- circular dependence

(b) Deadlock: no process holds all resources necessary to proceed. No work gets done.

Starvation: Some process never gets all resources necessary. Violation of fairness.

5. On a filesystem that allows soft links and hard links, you create `file_1`, soft-link `file_2` to `file_1`, and hard-link `file_3` to `file_2`. Then you delete `file_1`. Is there a way to still access the data? Explain. (5)

No, file_2 and file_3 will refer to the same soft link, the path pointing to file_1. But if file_1 is gone, file_2 and file_3 both appear as dangling soft-links.

6. RAID-10, also called RAID 1+0, is a RAID arrangement in which four (in the simplest case) disks are grouped in pairs. Each pair is operated as a RAID-1 ("mirroring") pair, the two pairs are then grouped together in a RAID-0 ("striping") configuration. Describe the characteristics of a RAID-10 configuration consisting of four identical disks, in the following categories:

   (a) storage capacity (relative to the capacity of a single disk),

   (b) protection against disk failure,

   (c) read and write performance (compare with bare RAID-0 and bare RAID-1).

(3+3+4)

(a) Effective capacity is half of the physical capacity of the four disks.

(b) Data will survive any single disk failure, and a double-disk failure if the failing disks do not belong to the same RAID-1 mirroring pair.

(c) Read speeds are as for a 4-disk RAID-0, thus very fast.

Write speed is as for a 2-disk RAID-0 (possibly just slightly slower as data needs to go to both mirror RAID-1 disks), so also rather good.

7. The following questions refer to the $(8,4)$-Hamming code with parity, using the bit ordering convention adopted in class.

   (a) Detect double-bit errors and correct single-bit errors in the following Hamming $(8,4)$-encoded bit streams: 00000111 and 10010101.

   (b) Encode the message 1011 as an $(8,4)$-Hamming code.

(5+5)

(a) $0\,0\,0\,\overset{d_1}{0}\,0\,\overset{d_2}{1}\,\overset{d_3}{1}\,\overset{d_4}{1}$

$p_0 = 1$, assume single-bit error

$$\left.\begin{array}{l} p_1 = 0 \\ p_2 = 0 \\ p_4 = 1 \end{array}\right\} \text{ error at } (100)_2 = 4 \quad (\text{parity bit } p_4)$$

$\Rightarrow$ message bits don't need correction, message is 0111.

$1\,0\,0\,\overset{d_1}{1}\,0\,\overset{d_2}{1}\,\overset{d_3}{0}\,\overset{d_4}{1}$

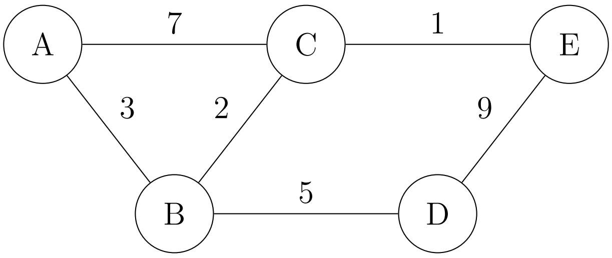
$p_0 = 0$, so overall parity is O.K.

$$\left.\begin{array}{l} p_1 = 1 \\ \vdots \end{array}\right\} \text{ indicates error.}$$

Since overall parity is O.K., error must be 2-bit, discard.

(b) $0\,0\,1\,1\,0\,0\,1\,1$

8. Consider the following router network, where the numbers on each edge indicate the cost of the link.



(a) Use distance-vector routing to compute the routing tables: Fill, for each router, a table of the form

Router A

Dest.	Next Hop	Cost
A	A	0
B	B	3
C	~~C~~ B	~~7~~ 5
D	~~-~~ B	~~∞~~ 8
E	~~-~~ B	~~∞~~ 6

Router B

Dest.	Next Hop	Cost
A	A	3
B	B	0
C	C	2
D	D	5
E	~~-~~ C	~~∞~~ 3

Router C

Dest.	Next Hop	Cost
A	~~A~~ B	~~7~~ 5
B	B	2
C	C	0
D	~~-~~ B	~~∞~~ 7
E	E	1

Router D

Dest.	Next Hop	Cost
A	~~-~~ B	~~∞~~ 8
B	B	5
C	~~-~~ B	~~∞~~ 7
D	D	0
E	~~E~~ B	~~9~~ 8

Round 1

Round 2

Router E

Dest.	Next Hop	Cost
A	~~-~~ C	~~∞~~ 6
B	~~-~~ C	~~∞~~ 3
C	C	1
D	~~D~~ C	~~9~~ 8
E	E	0

which you should update in steps until it no longer changes.

(b) Now the link C–E goes down. How many update steps does it take until the distance-vector algorithm has converged to the new solution?

(10+5)

- C-E will have a final cost of 16
- In each update cycle, C-E will be two worse than B-E,
  B-E will be two worse than C-E
- Thus, the cost C-E goes up 4 in a complete update cycle.
- Thus, to go from 1 to 16 in steps of 4, 4 complete update cycles are required.

(This is an example of "bad news travels slow" ...)