Foundations of Information Systems

Final Exam

February 21, 2023

1. Determine whether the following statements are true or false. If true, state the name of the respective Boolean algebra axiom or elementary theorem (a proof is not required). If false, state a counterexample.

   (a) $a \wedge b = b \wedge a$

   (b) $a \vee 0 = 0$

   (c) $a \wedge (b \vee c) = (a \vee b) \wedge (a \vee c)$

   (d) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

   (e) $(a \vee b)' = a' \wedge b'$

   (2+2+2+2+2)

2. (a) Interpret the hexadecimal number 0xCB as an 8-bit two's complement signed integer and convert to decimal.

   (b) Perform the following binary addition:

   $$0110\,1111 + 0110\,1010$$

   (c) Does the result of the computation in (b) cause an overflow if the two numbers are interpreted as (i) 8-bit unsigned integers and (ii) as 8-bit two's complement signed integers?

   (4+3+3)

3. (a) Perform the following computation in decimal floating point with five significant digits:

   $$(1.0000 \cdot 10^0 + 3.3333 \cdot 10^{-3}) - 1.0000 \cdot 10^0$$

   Suggest a different ordering of arithmetic operations that improves the accuracy of the computation.

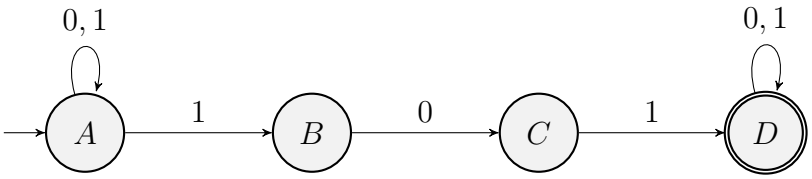
   (b) Consider the evaluation of the following expression in floating point arithmetic:

   $$\frac{1 - (1 - x)^3}{x}$$

   Identify values of x for which there is a substantial growth of relative error and suggest an alternate formula that improves accuracy for the problematic range of x.

   (5+5)

4. Consider the following FSA:



   (a) Which strings are recognized by this FSA? Give a plain language answer and also state the corresponding regular expression.

   (b) Convert this nondeterministic FSA into a deterministic FSA.

   (5+5)

5. (a) Process P_1 holds resource R_1 and needs resource R_2. Process P_2 holds resource R_2 and needs resource R_3. Process P_3 holds resource R_3 and needs resource R_1. Draw a resource allocation graph and argue that this is a deadlock situation.

   (b) Now impose the following ordering constraint: *A process may hold resource R_i only if it already holds all R_j with $j < i$ that it needs.*

   Illustrate that this rule breaks the deadlock situation in (a). Does this rule prevent deadlock in general? Does it prevent starvation in general?

   (5+5)

6. On a filesystem that allows soft links and hard links, you create `file_1`, hard-link `file_2` to `file_1`, and soft-link `file_3` to `file_1`. Then you delete `file_1`. What happens if you access `file_2`? What happens if you access `file_3`? Explain. (5)

7. (a) In a RAID-5 array, one disk out of 4 has failed. The others contain the bit sequences $0000\,1111\ldots$, $0101\,0101\ldots$, $0100\,0010\ldots$. Reconstruct the beginning of the bit sequence of the failed disk.

   (b) An IBAN is checked by verifying that, after suitable rearrangement, replacement of letters by number digits, and conversion to integer, the given number modulo 97 equals 1. What is the rationale behind the convention that 1 rather than 0 indicates correctness?

   (5+5)

8. The $(3, 1)$-Hamming code is the shortest code in the Hamming family of codes.

   (a) Explain the construction of the 3-bit codeword from the single data bit. Then explain how a single-bit error is corrected.

   (b) Draw a truth table which describes the mapping from arbitrary 3-bit messages (valid code-words possibly corrupted by single-bit errors) to the *corrected* single data bit.

   (c) Convert the truth table from part (b) into a Boolean algebra expression, simplify if possible, then draw the resulting Boolean function as an array of logic gates.

(d) *(Extra credit.)* Draw an FSA that reads three data bits of a $(3, 1)$-Hamming-encoded message and has two final states, 1 and 0, representing the *corrected* single data bit.

(5+5+5+*5*)