Stochastic Processes

Summer Semester 2026, Exercise 2

Due Thursday, May 7, 2026

1. Recall the classical *Monty Hall problem*:

   > Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

   Now suppose that it has been observed that Monty Hall, the host, has a preference for door #2: whenever it is possible to open door #2 without showing the car, he will open this door.

   Given this additional information, is it to your advantage to switch if he opens door #2 and you see a goat? Is it to your advantage if he opens door #3 and you see a goat? Perform a quantitative analysis of this variation in terms of conditional probabilities.

2. (Humpherys & Jarvis, Problem 5.7) Five of my friends come to dinner and take their coats off at the door when they arrive, and my ever-helpful son puts the coats away in his room. The guests leave one at a time, and when each one leaves, my son brings back a random coat (selected uniformly) and gives it to them. Since the guests are in a hurry, they each put on the coat given to them without noticing whether it is correct, and then they leave.

   (a) What is the probability that the first guest gets the right coat?

   (b) If the first to leave gets the right coat, what is the probability that the second to leave will get the right coat?

   (c) What is the probability that every guest will get the right coat?

   (d) If the first guest to leave gets the coat belonging to the second guest, what is the probability that the second guest will get the right coat?

   (e) If the first coat is wrong, but it is also not the second guest's coat, what is the probability that the second guest will get the right coat?

   (f) Without knowing the outcome of the first coat, what is the probability that the second coat will be right? Hint: consider using the law of total probability.

3. (Humpherys & Jarvis, Problem 5.17) Let X be the outcome of the roll of a fair six-sided die. Find the expectation and the variance of X.

4. (Humpherys & Jarvis, Problem 5.30) A biologist is collecting kangaroo rats in the desert and she is hoping to find some with a certain trait that occurs in 10% of the general population of kangaroo rats. She collects 100 rats in total. Assuming that each sample is independent of the others:

   (a) Which distribution describes the exact number of rats that have the trait?

   (b) What is the expected number of rats that will have the trait?

   (c) What is the probability that she will find exactly 30 rats with the trait?

   Note: For (b) and (c), you may submit a Python program with output. (Is it possible to get a quick pencil-and-paper answer?)