Models of Weather and Climate

Models of geophysical fluid flow – review questions and problems

Summer Semester 2025

1. What is the meaning of the "material derivative" D_t? Can you show that $D_t = \partial_t + u \cdot \nabla$? Which physical quantities θ, in typical geophysical models, satisfy $D_t\theta = 0$ ("Material conservation of θ")?

2. How can you write Newton's second law for a fluid? What additional forces (per unit area or per unit volume) are particularly relevant for large-scale geophysical flow?

3. Conservation laws for geophysical fluid flow:

   (a) Material conservation of potential vorticity. For the shallow water equation in non-dimensional variables, the potential vorticity is

   $$q = \frac{1 + \mathrm{Ro}\ \nabla^\perp \cdot u}{H_0 + h}\,. \tag{1}$$

   Can you interpret/explain the physical significance?

   (b) Conservation of volume (or "incompressibility"). How do you derive the condition $\nabla \cdot u = 0$? (Being able to give a detailed derivation would be great, but you should at least know a sketch of the proof.)

   (c) Conservation of mass. How do you derive the "continuity equation"

   $$\partial_t \rho + \nabla \cdot (\rho u) = 0\,? \tag{2}$$

   (d) Conservation of total energy for fluid flow without viscosity and forcing. E.g., for the shallow water equations written in the form

   $$\mathrm{Ro}\,(\partial_t u + u \cdot \nabla u) + u^\perp + \frac{\mathrm{Bu}}{\mathrm{Ro}}\,\nabla h = 0\,, \tag{3a}$$

   $$\partial_t h + \nabla \cdot (hu) + H_0\,\nabla \cdot u = 0\,, \tag{3b}$$

   can you show that the total energy

   $$E = \frac{1}{2}\int_\Omega \mathrm{Ro}\,(H_0 + h)\,|u|^2 + \frac{\mathrm{Bu}}{\mathrm{Ro}}\,h^2\,\mathrm{d}x \tag{4}$$

   is conserved?

(e) Local conservation laws for momentum (i.e., Newton's second law) and energy: we did not discuss local conservation laws systematically, but an idea of the conceptual distinction between "global conservation law", "material conservation law", "local conservation law" is helpful.

(f) If you add a viscous force, i.e. $\nu \Delta u$, to the right hand side of the momentum equation, what does this do to the evolution of total energy?

4. Geostrophic balance:

   (a) What is it and why is it important?

   (b) Can you show that in planar geometry, e.g. in the context of the shallow water system (**??**) above, a solution in geostrophic balance is stationary?

   (c) Does geostrophic balance hold near the equator?

5. Linear waves: Inertia-gravity waves vs. Rossby waves. Inertia-gravity waves are dispersive. What does this mean? What is the physical significance of the shallow water dispersion relation (in dimensional variables)

$$\omega(k) = \sqrt{f^2 + g\,H\,|k|^2}$$

6. Baroclinic instability: What is it and why is it important? In the Eady model of baroclinic instability, what is the assumed steady-state background flow? What are the assumed background density surfaces? How are the unstable modes calculated? What is the role of baroclinic instability for the global energy balance of atmosphere and ocean?

7. Non-dimensionalization and scaling: when written in the nondimensionalized form (**??**), which was derived by non-dimensionalizing all variables, the characteristic scale height of h is not yet specifified. Can you explain why there is still one more scaling parameter to choose? What are possible choices given that a dominant balance of pressure gradient and Coriolis term is consistent with observations (at large scales at the mid-latitudes)?

8. Hydrostatic balance: what is it? Why is it often imposed, even in operational models? What is the underlying scaling limit that leads to hydrostaticity?