# Differential Equations

Summer Semester 2024, Exercise 4

Due Wednesday, May 15, 2024

1. Find all equilibrium points, determine their stability properties, and try to sketch the phase portraits of the following differential equations:

   (a) $\dot{x}_1 = x_2\,(1 + x_1 - x_2^2)$
   $\dot{x}_2 = x_1\,(1 + x_2 - x_1^2)$

   (b) $\dot{x}_1 = 2x_1 - x_1^2 - x_1 x_2$
   $\dot{x}_2 = -x_2 + x_1 x_2$

   (c) $\ddot{y} + \dot{y} + y^3 = 0$

2. Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1^3 + \mu\, x_1\,, \\ \dot{x}_2 &= -x_1\,. \end{aligned}$$

   (a) For which values of the parameter $\mu$ does a periodic solution exist?

   (b) Describe what happens as $\mu \searrow 0$.