# Algorithms and Data Structures

Summer Semester 2024

For discussion on Wednesday, June 12, 2022

1. Give a proof of the index-relationships for an array-based implementation of a binary tree:

   (a) $i(l) = 2i(p) + 1$ if $l$ is the left child of $p$,

   (b) $i(r) = 2i(p) + 2$ if $r$ is the left child of $p$,

   (c) $i(p) = \lfloor (i(c) - 1)/2 \rfloor$ if $p$ is the parent of $c$.

2. (GTG Exercise R-9.2) Suppose you label each position $p$ of a binary tree $T$ with a key equal to its preorder rank. Under what circumstances is $T$ a heap?

3. (GTG Exercise R-9.3) What does each remove min call return within the following sequence of priority queue ADT methods:

   `add(5,A)`, `add(4,B)`, `add(7,F)`, `add(1,D)`, `remove_min()`, `add(3,J)`, `add(6,L)`, `remove_min()`, `remove_min()`, `add(8,G)`, `remove_min()`, `add(2,H)`, `remove_min()`, `remove_min()`?

4. (GTG Exercise R-9.10) At which positions of a heap might the third smallest key be stored?

5. (GTG Exercise R-9.11) At which positions of a heap might the largest key be stored?

6. (GTG Exercise R-9.13) Illustrate the execution of the in-place heap-sort algorithm on the following input sequence: (2, 5, 16, 4, 10, 23, 39, 18, 26, 15).

7. (GTG Exercise R-9.16) Is there a heap $H$ storing seven entries with distinct keys such that a preorder traversal of $H$ yields the entries of $H$ in increasing or decreasing order by key? How about an inorder traversal? How about a postorder traversal? If so, give an example; if not, say why.

8. (GTG Exercise R-9.18) Show that the sum

$$\sum_{i=1}^{n} \log i$$

   which appears in the analysis of heap-sort, is $\Omega(n \log n)$.