Review of first example from class on April 1:

5 cards are drawn from standard deck

event E: "two pairs": have two cards of one value
" " another value
one card of a third value } 3 values

1. Choose these 3 values from 13 available values

$$C^{13}_3$$

2. From the three chosen values, select one which is "single".

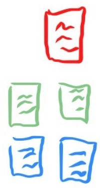
$$C^3_1$$

1, 2, 2

3. From "single" value, draw one card: C^4_1

4. From each of the "double" values, draw two cards

$$C^4_2 \cdot C^4_2$$



$\Rightarrow$ Total count is $C^{13}_3 \cdot C^3_1 \cdot C^4_1 \cdot C^4_2 \cdot C^4_2$

Remark: $C^3_2 = C^3_1 = \dfrac{3!}{1!\,2!}$

Review examples:

① you have 4 keys in your pocket, only one fits your door.

Q: What is the probability to open door in at most 3 trial ?

Easier question: what is the probability to need four attempts ?

- correct key is last, with prob. $\frac{1}{4}$

So answer to the original question: $p = 1 - \frac{1}{4} = \frac{3}{4}$

Long answer:

$P(\text{first key works}) = \frac{1}{4}$

$P(\text{success in } 2^{nd} \text{ attempt}) = \underbrace{P(\text{first key didn't work})}_{\frac{3}{4}} \underbrace{P(\text{second works} \mid \text{first key didn't})}_{\frac{1}{3}} = \frac{1}{4}$

$P(\text{success in } 3^{rd} \text{ attempt}) = \underbrace{P(\text{didn't work in first two attempts})}_{\frac{1}{2}} \underbrace{P(\text{works in } 3^{rd} \mid \text{didn't work earlier})}_{\frac{1}{2}}$

$= \frac{1}{4}$

In total, with prob. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$, we are successful within 3 attempts.

total prob. $\frac{3}{4}$

② "Birthday problem"

60 students : what is the prob. that two students have their birthday on the same day? A

First, look at probability of not finding two students who have the same birthday: B

$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{365-59}{365} = 0.00017$$

$$P(\text{two birthdays on same day}) = 1 - 0.00017 = 99.983\,\%$$

A, B mutually exclusive: $P(A \cup B) = P(A) + P(B)$

$S = A \cup B$ $\qquad P(S) = \boxed{1 = P(A) + P(B)} \Rightarrow P(A) = 1 - P(B)$

Bayes' Rule

Recall:
$$P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)$$

$P(B) = P(B|A)$ iff A, B independent

$$\Rightarrow P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

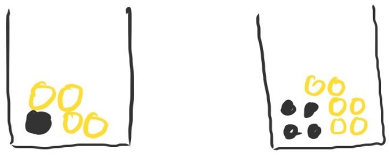
"posterior" likelihood "prior"

$$P(A|B) = \frac{P(B|A)}{P(B)} \cdot P(A)$$

marginalization

Examples:

① Jar 1: one black, 4 white marbles
" 2: 4 " , 6 " "



J1 J2

One jar is chosen at random (tossing a coin), then a marble is chosen

B: event that chosen marble is black
J1: " " marble comes from jar 1
J2: " " " " " , 2

$$P(B) = P(B|J1)\,P(J1) + P(B|J2)\,P(J2)$$
$$= \frac{1}{5}\cdot\frac{1}{2} + \frac{4}{10}\cdot\frac{1}{2} = \frac{3}{10}$$

$$P(J1|B) = \frac{P(B|J1)\cdot P(J1)}{P(B)} = \frac{\frac{1}{5}\cdot\frac{1}{2}}{\frac{3}{10}} = \frac{1}{3}$$

② Drug test

97% sensitive (true positives)

95% specific (true negatives)

$0.5\% = \frac{1}{200}$

Know: 0.5% of population uses the drug.

A randomly selected person tests positive, what is the probability that the person takes the drug?

$\underbrace{\text{tests positive}}_{T}$, $\underbrace{\text{takes the drug}}_{D}$

$$P(D|T) = \frac{P(T|D)\,P(D)}{P(T)}$$

$$P(T) = \underbrace{P(T|D)}_{\frac{97}{100}}\underbrace{P(D)}_{\frac{1}{200}} + \underbrace{P(T,\bar{D})}_{\frac{5}{100}}\underbrace{P(\bar{D})}_{\frac{199}{200}}$$

$$= \frac{\frac{97}{100\cdot\cancel{200}}}{\frac{1092}{\cancel{100}\cdot\cancel{200}}} = \frac{97}{1092} \approx 9\% \qquad = \frac{1092}{100\cdot 200}$$