Finite Mathematics

Final Exam Review

Date of exam: May 27, 2020, 16:00–18:00

1. All linear algebra topics covered up to the mock midterm exam. Note that solving a system of equations is part of the solution process to find equilibrium properties for regular Markov chains; inversion of a matrix is required in the context of absorbing Markov chains.

   Least square solutions and/or least norm solutions are important; you should expect at least one of them on the final exam.

2. Basic notions from set theory, counting (multiplication principle, permutations, combinations): important to review, may not be tested directly, but as part of a probability problem.

   (Homework 6 and Homework 7, Questions 1 and 2.)

3. Basic notions from probability: mutually exclusive events, addition rule, conditional probability, independent events.

   All the Chapter 14 "Chapter Review" questions from the CNX notes are good review problems. There will similar questions on the final exam.

4. Bernoulli trials (binomial probability), expected value.

   The Chapter 15 "Chapter Review" questions from the CNX notes provide plenty of practice material.

5. Bayes' formula.

   Example problem: There are two types of students. Students of type A report sick only when they are actually sick. Students of type B report sick with probability 1 whenever there is an exam. It is known that the sickness rate of the general working population is 5%. On every exam, 10% of students report sick.

   (a) What is the probability that a randomly selected student is of type B?

   (b) What is the probability that a student who reports sick on the day of the exam is of type B?

   (c) A student reports sick on the midterm and on the final. What is the probability that the student is of type B?

(This is an old exam problem from the final exam of "Engineering and Science Mathematics 2B" from Spring 2013. You can find a worked-out answer on my web page. Problem 6 from that same final exam provides another nice Bayes' rule problem.)

6. Regular Markov chains: Check whether a Markov chain is regular. Find the vector of equilibrium probabilities by solving a linear system of equations.

   Example problem: A truck driver distributes goods between warehouses in Amsterdam, Berlin, and Copenhagen. For a tour starting in Amsterdam, there is a 30% chance that it'll end in Amsterdam again, a 30% chance that it'll end in Berlin, and a 40% chance that it'll end in Copenhagen. For a tour starting in Berlin, there is a 40% chance that it'll go to Amsterdam, a 40% chance that it'll remain in Berlin, and a 20% chance that it'll go to Copenhagen. For a tour starting in Copenhagen, there is a 50% chance that it'll go to Amsterdam, a 30% chance that it'll go to Berlin, and a 20% chance to remain in Copenhagen. Assuming, for simplicity, that each tour will be done in a day, what is the probability that the driver will spend the night in each of the three cities?

   (I will publish a sample solution on Monday, May 18.)

7. Absorbing Markov chains: Recognize an absorbing Markov chain. Write the transition matrix in canonical form, solve for the fundamental matrix and for the matrix of absorbing probabilities. Find the mean exit times.

   Typical problems: Homework 9, Question 5.

8. Two-player zero-sum games: Be able to check whether a game with a given pay-off matrix is strictly determined. If not, be able to perform reduction-by-dominance if necessary, find the optimal mixed strategies for both players, and determine the value of the game.

   Typical problems: Homework 10, Question 3.