Finite Mathematics

Homework 5

Due in class Monday, March 16, 2020

1. Recall that if the columns of an $m \times n$ matrix with $m \geq n$ (the matrix is taller than wide) are linearly independent, the least-square solution to the potentially over-determined linear system $A\boldsymbol{x} = \boldsymbol{b}$ is given by

$$\boldsymbol{x} = (A^T A)^{-1} A^T \boldsymbol{b}\,.$$

   Show that this expression is equivalent to solving the linear system

$$\begin{aligned} A^T \boldsymbol{y} &= 0\,, \\ A\boldsymbol{x} + \boldsymbol{y} &= \boldsymbol{b}\,. \end{aligned}$$

2. Find the least square solution of the system

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \boldsymbol{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\,.$$

3. Find the best straight line fit (in the least-square sense) to the points $(0, 3)$, $(1, 3)$, and $(1, 6)$.

4. Find the best plane fit (in the least-square sense) to the points $(1, 1, 1)$, $(0, 1, 0)$, $(1, 0, 2)$, and $(0, 0, 0)$.

   Hint: Describe the plane by the equation $z = a + b\,x + c\,y$ and write out 4 equations in the 3 unknowns a, b, and c for the plane to pass through these points. Then find the least-square solution of this over-determined system of equations.