Finite Mathematics

Mock Midterm Exam

March 24, 2020

1. Find a parametric description of the plane which goes through the points

$$\boldsymbol{a} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}, \qquad \boldsymbol{c} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

(5)

2. Mark the following statements as *always true*, *sometimes true*, or *never true*.

   (a) If the augmented matrices of two systems of linear equations are row equivalent (i.e., can be transformed into each other by elementary row operations), then both systems have the same solution set.

   (b) To multiply one row of the augmented matrix by a constant is an elementary row operation.

   (c) Any linear system with n equations and n unknowns can have at most n solutions.

   (d) If a system of linear equations has two different solutions, then it has infinitely many solutions.

   (e) A homogeneous system of linear equations can be inconsistent.

   (f) A homogeneous system of linear equations has infinitely many solutions.

   (g) $A\boldsymbol{x} = \boldsymbol{b}$ has infinitely many solutions if the columns of A are linearly dependent.

   (h) If A is a 4×10 matrix, the system $A\boldsymbol{x} = \boldsymbol{b}$ has a unique solution.

   (i) If A is a 10×4 matrix, the system $A\boldsymbol{x} = \boldsymbol{b}$ has a unique solution.

   (j) If A is an invertible square matrix, the system $A\boldsymbol{x} = \boldsymbol{b}$ has a unique solution.

(10)

3. Let

$$A = \begin{pmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -4 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

   (a) Find the general solution to the linear system $A\boldsymbol{x} = \boldsymbol{b}$.

(b) Find the least-norm solution to the linear system $A\boldsymbol{x} = \boldsymbol{b}$.

(5+5)

4. Which of the pairs of the following vectors are perpendicular, which ones are not?

$$\boldsymbol{u} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \qquad \boldsymbol{w} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

(5)

5. Consider the plane with parametric representation

$$\boldsymbol{x} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}.$$

Given the points

$$\boldsymbol{p} = \begin{pmatrix} -5 \\ -1 \\ 7 \end{pmatrix}, \qquad \boldsymbol{q} = \begin{pmatrix} 3 \\ 32 \\ 6 \end{pmatrix},$$

show that one of the points $\boldsymbol{p}$ or $\boldsymbol{q}$ lies in the plane, the other one does not.

In each case, compute the values of the parameters λ and μ which correspond to the given point in the plane, resp. to the point in the plane closest to the given point. (15)

6. Show that if A has linearly independent columns, then $A^T A$ is invertible. (5)