# Analysis II

Review Sheet for the Final Exam

Exam takes place May 31, 2019, 9:00-11:00, Lecture Hall Research III

The final exam will cover:

- Any topic from the midterm review problem set.
- Differentiation in metric spaces. Review, in particular, Homework 7 Question 2.
- Inverse and implicit function theorems. Review the relevant questions from the homework, in particular Homework 10 Question 2 and Question 1 below.
- Lagrange multipliers. See Homework 11 and Question 2 below.
- Iterated integrals. See Homework 11.
- Change-of-variable formula. See Homework 12 and Question 3 below.
- Divergence theorem. See Homework 12 and Question 4 below.

## Additional practice problems

1. Let $A \subset \mathbb{R}^n$ be open, $f\colon A \to \mathbb{R}^n$ continuously differentiable and injective, and $f'(x)$ invertible for every $x \in A$. Prove the following.

   (a) $f(A)$ is open,

   (b) $f^{-1}$ is differentiable on $f(A)$,

   (c) for every $B \subset A$ open, $f(B)$ is open.

   *Hint:* inverse function theorem.

2. Let $p$ and $q$ be positive real numbers with

$$\frac{1}{p} + \frac{1}{q} = 1\,.$$

(a) Maximize

$$f(x,y) = x^{1/p}\, y^{1/q}$$

subject to $x, y \geq 0$ and

$$g(x,y) = \frac{x}{p} + \frac{y}{q} = c$$

with $c > 0$ fixed.

(b) Conclude that, for $x, y \geq 0$ and $p, q$ as above,

$$x^{1/p}\, y^{1/q} \leq \frac{x}{p} + \frac{y}{q}\,.$$

3. Use the change of variables formula to compute the area of the image of the unit square under the map

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + y \\ y^2 - x \end{pmatrix}\,.$$

4. Compute the flux

$$\int_{\partial D} \boldsymbol{F} \cdot \boldsymbol{n}\, \mathrm{d}S\,,$$

where $\boldsymbol{n}$ is the outward unit normal and

$$\boldsymbol{F} = \begin{pmatrix} x^2 \\ x^2\, y \\ -x^2\, z \end{pmatrix}$$

through the surface of the tetrahedron $D$ with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

(a) Use the divergence theorem.

(b) Compute the flux integral directly by using a suitable parametrization of the surface integral.