Applied Differential Equations and Modeling

Homework 9

Due in class Tuesday, April 23, 2019

1. Find the solution to the following initial value problems.

   (a) $y'' + 4\,y = t^2 + 3\,e^t\,, \quad y(0) = 0\,, \quad y'(0) = 2$

   (b) $y'' - 2y' - 3y = 3t\,e^{2t}\,, \quad y(0) = 1\,, \quad y'(0) = 0$

2. Find the general solution to the initial value problem

$$u'' + \omega_0^2\, u = \cos\omega t$$

   for

   (a) $\omega \neq \omega_0$,

   (b) $\omega = \omega_0$.

3. Consider the equation of a damped-driven oscillator,

$$y'' + 0.25\, y' + 2\, y = 2\, \cos\omega t\,.$$

   (a) Find the gain function $|G(\mathrm{i}\omega)|$ for this problem.

   (b) For which value of ω is the the gain maximal? Is this value smaller or larger than the frequency ω_0 of the free, undamped equation?

   (c) Solve the equation with initial values $y(0) = 0$ and $y'(0) = 2$.

4. Consider a constant coefficient second order equation with inhomogeneous right hand side, i.e.

$$a\, y'' + b\, y' + c\, y = g(t)\,. \qquad (*)$$

   Show that if the characteristic equation

$$a\,\lambda^2 + b\,\lambda + c = 0$$

   has two roots with negative real part, then all solutions to the differential equation coincide asymptotically. In other words, if y_1 and y_2 are two solutions of (*), then

$$\lim_{t\to\infty} (y_1(t) - y_2(t)) = 0\,.$$