Applied Differential Equations and Modeling

Homework 7

Due in class Tuesday, March 26, 2019

1. For each of the following systems of nonlinear differential equations,

   - find all equilibrium points,
   - for each equilibrium point, write out the linear system describing the evolution of small perturbations about the equilibrium points, compute its eigenvalues and, if applicable, eigenvectors, and
   - sketch the phase portrait to the extent possible.

   (a) $x' = x - x\,y$
   $y' = y + 2\,x\,y$

   (b) $x' = 2 - x$
   $y' = y - x^2$

   (c) $x' = -(x - y)\,(1 - x - y)$
   $y' = x\,(2 + y)$

2. Consider a general homogeneous constant-coefficient second-order linear differential equation, i.e.,

$$a\,y'' + b\,y' + c\,y = 0\,.$$

   (a) Convert this equation into a system of linear first-order equations with matrix A.

   (b) Show that

   - when $b^2 - 4ac > 0$, the eigenvalues of A are real and distinct,
   - when $b^2 - 4ac = 0$, A has one eigenvalue of multiplicity 2,
   - when $b^2 - 4ac < 0$, A has a pair of complex conjugate eigenvalues.

   (c) Find the general solution to the equation

$$y'' - 2\,y' + 2\,y = 0$$

   in terms of real-valued functions.