Applied Differential Equations and Modeling

Homework 6

Due in class Tuesday, March 19, 2019

1. For each of the following matrices,

   - find the eigenvalues and eigenvectors, if necessary a generalized eigenvector,
   - write out the general solution for the homogeneous linear differential equation
   $$\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}\,,$$
   - sketch the phase portrait of the differential equation.

   (a) $A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

2. For each of the following problems, find the solution of the given initial value problem. Draw the trajectory of the solution in the x_1-x_2 plane and also draw the component plots of x_1 versus t and of x_2 versus t.

   (a) $\boldsymbol{x}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \boldsymbol{x}\,, \qquad \boldsymbol{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

   (b) $\boldsymbol{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \boldsymbol{x}\,, \qquad \boldsymbol{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

3. For each of the following systems of differential equations,

   - find the equilibrium solution,
   - describe how the system behaves in the vicinity of the equilibrium solution,
   - sketch the phase portrait.

   (a) $x' = -x + y + 1\,,$
   $y' = x + y - 3$

(b) $x' = -x - 4\,y - 4\,,$
$y' = x - y - 6$

4. Describe how the nature of the eigenvalues depends on the parameter α in the matrix

$$A = \begin{pmatrix} 2 & \alpha \\ 1 & -3 \end{pmatrix}\,.$$