Applied Differential Equations and Modeling

Homework 5

Due in class Tuesday, March 12, 2019

1. Compute the determinant and the inverse, if possible, of each of the following matrices.

   (a) $\boldsymbol{A} = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$

   (b) $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$

   (c) $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 8 \\ 1 & -2 & -7 \end{pmatrix}$

2. Find the general solution of the given set of equations, or else show that there is no solution.

   (a) $\begin{aligned} 2\,x_1 + x_2 + x_3 &= 2 \\ -\,x_1 + x_3 &= 1 \\ x_1 + x_2 + 2\,x_3 &= 3 \end{aligned}$

   (b) $\begin{aligned} 2\,x_1 + x_2 + x_3 &= 0 \\ -\,x_1 + x_3 &= -1 \\ x_1 + x_2 + 2\,x_3 &= 1 \end{aligned}$

   (c) $\begin{aligned} x_1 - x_2 + x_3 + x_4 &= -1 \\ x_2 + x_3 + 3\,x_4 &= 2 \\ x_1 + 2\,x_3 + 4\,x_4 &= 1 \\ x_2 + x_3 + 3\,x_4 &= 2 \end{aligned}$

3. Find all eigenvalues and eigenvectors for each of the following matrices. If possible, find a matrix $\boldsymbol{S}$ and a diagonal matrix $\boldsymbol{D}$ such that

$$\boldsymbol{A} = \boldsymbol{S}\boldsymbol{D}\boldsymbol{S}^{-1}\,.$$

(a) $\boldsymbol{A} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$

(b) $\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$

(c) $\boldsymbol{A} = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$