Applied Differential Equations and Modeling

Homework 4

Due in class Tuesday, March 5, 2019

1. Given the matrices

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 \\ -1 & 3 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} -1 & 1 & -2 \\ 1 & 3 & 4 \end{pmatrix}, \quad \boldsymbol{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 1 & 2 & -1 \end{pmatrix},$$

compute the following. Note that not all operations may be well-defined.

(a) $\boldsymbol{A} + \boldsymbol{B}$

(b) $\boldsymbol{AB}$

(c) $\boldsymbol{BA}$

(d) $\boldsymbol{Ax}$

(e) $\boldsymbol{Bx}$

(f) $\boldsymbol{B}^T\boldsymbol{A}$

(g) $\boldsymbol{B}^T\boldsymbol{A}^T$

(h) $\boldsymbol{y}\boldsymbol{B}^T$

(i) $\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x}$

(j) $\boldsymbol{x}\boldsymbol{B}^T\boldsymbol{y}^T$

2. Demonstrate that

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -1 & 1 \\ 1 & 0 & -2 \end{pmatrix}$$

are both nonsingular by showing that $\boldsymbol{AB} = \boldsymbol{I}$.

3. Prove the following:

(a) If $\boldsymbol{A}$ is symmetric and nonsingular, then $\boldsymbol{A}^{-1}$ is symmetric.

(b) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric, then $\boldsymbol{AB}$ is symmetric if and only if $\boldsymbol{AB} = \boldsymbol{BA}$. (We then say that "$\boldsymbol{A}$ and $\boldsymbol{B}$ commute".)

(c) $\boldsymbol{A}\boldsymbol{A}^T$ is symmetric.

(d) If $\boldsymbol{A}$ is a square matrix, then $\boldsymbol{A} + \boldsymbol{A}^T$ is symmetric.

4. In each case, reduce $\boldsymbol{A}$ to row echelon form and determine rank $\boldsymbol{A}$.

(a) $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{pmatrix}$

(b) $\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(c) $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}$

5. In each of the following, if there exist solutions of the homogeneous system of linear equations other than $\boldsymbol{x} = \boldsymbol{0}$, express the general solution as a linear combination of linearly independent column vectors.

(a) $x_1 - x_3 = 0$
$3\,x_1 + x_2 + x_3 = 0$
$-\,x_1 + x_2 + 2\,x_3 = 0$

(b) $x_1 - 2\,x_2 + x_4 = 0$
$2\,x_1 + x_2 + x_3 - x_4 = 0$
$x_1 + 2\,x_2 + x_3 - 2\,x_4 = 0$
$3\,x_1 + 3\,x_2 + 2\,x_3 - 3\,x_4 = 0$

6. Determine whether the members of the given set of vectors are linearly independent. If they are linearly dependent, find a linear relation among them.

(a) $\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \boldsymbol{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(b) $\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad \boldsymbol{v}_2 = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{v}_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \boldsymbol{v}_4 = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$