Applied Differential Equations and Modeling

Homework 12

Due in class Wednesday, May 15, 2019

1. Find the solution of the given initial value problem and draw its graph.
   (a) $y'' + 2y' + 2y = \delta(t - \pi)$ with $y(0) = 1$, $y'(0) = 0$
   (b) $y'' + y = \delta(t - 2\pi)\cos t$ with $y(0) = 0$, $y'(0) = 1$
2. Find the Laplace transform of the given function.
   (a) $\displaystyle\int_0^t (t - \tau)^2 \cos 2\tau \, \mathrm{d}\tau$
   (b) $\displaystyle\int_0^t \mathrm{e}^{-(t-\tau)} \sin \tau \, \mathrm{d}\tau$
3. Find the inverse Laplace transform using the convolution theorem.
   (a) $F(s) = \dfrac{1}{s^4 (s^2 + 1)}$
   (b) $F(s) = \dfrac{G(s)}{s^2 + 1}$
4. In each of the following problems, express the total response in terms of the forced response (using a convolution integral) and the free response.
   (a) $y'' + \omega^2 y = g(t)$ with $y(0) = 0$, $y'(0) = 1$
   (b) $y'' + 3y' + 2y = \cos(\alpha t)$ with $y(0) = 1$, $y'(0) = 0$
5. Which of the following transfer functions corresponds to a BIBO-stable system?
   (a) $H(s) = \dfrac{1}{(s + 1)^2 (s^2 + 1)}$
   (b) $H(s) = \dfrac{1}{s^2 - 1}$
   (c) $H(s) = \dfrac{1}{6 + 11s + 6s^2 + s^3}$