Applied Differential Equations and Modeling

Homework 6

Due in class Tuesday, April 17

1. Find the solution to the following initial value problems.

   (a) $y'' + 4\,y = t^2 + 3\,e^t\,, \quad y(0) = 0\,, \quad y'(0) = 2$

   (b) $y'' - 2y' - 3y = 3t\,e^{2t}\,, \quad y(0) = 1\,, \quad y'(0) = 0$

2. Find the general solution to the initial value problem

   $$u'' + \omega_0^2\,u = \cos\omega t$$

   for

   (a) $\omega \neq \omega_0$,

   (b) $\omega = \omega_0$.

3. Find the gain function $|G(\mathrm{i}\omega)|$ for the vibrating system described by the initial value problem

   $$y'' + 0.25\,y' + 2\,y = 2\,\cos\omega t\,, \quad y(0) = 0\,, \quad y'(0) = 2\,.$$

   For which value of ω is the the gain maximal? Is this value smaller or larger than the resonance frequency of the undamped equation?

4. Consider a constant coefficient second order equation with inhomogeneous right hand side, i.e.

   $$a\,y'' + b\,y' + c\,y = g(t)\,. \qquad (*)$$

   Show that if the characteristic equation

   $$a\,\lambda^2 + b\,\lambda + c = 0$$

   has two roots with negative real part, then all solutions to the differential equation coincide asymptotically. In other words, if y_1 and y_2 are two solutions of (*), then

   $$\lim_{t\to\infty}(y_1(t) - y_2(t)) = 0\,.$$