Applied Differential Equations and Modeling

Homework 5

Due in class Thursday, March 22, 2018

1. Find the (single) eigenvalue, eigenvector $\boldsymbol{v}$, and generalized eigenvector $\boldsymbol{w}$ for each of the following matrices.

   (a) $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

   (b) $A = \begin{pmatrix} -\frac{3}{2} & 1 \\ -\frac{1}{4} & -\frac{1}{2} \end{pmatrix}$

2. For the matrices from Problem 1, write out the general solution ot the system $\dot{\boldsymbol{x}} = A\boldsymbol{x}$. Describe how the solutions behave as $t \to \infty$.

3. For the matrices from Problem 1, find the solution to $\dot{\boldsymbol{x}} = A\boldsymbol{x}$ which corresponds to the initial condition $x(0) = (1, 1)$.

4. Consider linear equations of the form

$$\dot{\boldsymbol{x}} = A\boldsymbol{x}\,,$$

   where A is a 2×2 matrix with *real* coefficient. Is the following true or false? If a short argument for each case.

   (a) A can have repeated complex eigenvalues (i.e., eigenvalues with a non-zero imaginary part).

   (b) If none of the eigenvalues of A has a strictly positive real part, then there is no solution which grows in time (i.e., then the trivial solution $\boldsymbol{x}(t) = \mathbf{0}$ is stable).