Applied Differential Equations and Modeling

Homework 4

Due in class Tuesday, March 13, 2018

1. Find the eigenvalues and eigenvectors for each of the following matrices.

   (a) $A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

2. For the matrices from Problem 1, sketch the phase portrait for the corresponding linear differential equation $\dot{\boldsymbol{x}} = A\boldsymbol{x}$.

3. Describe how the nature of the eigenvalues depends on the parameter α in the matrix

$$A = \begin{pmatrix} 2 & \alpha \\ 1 & -3 \end{pmatrix} .$$

4. Verify that the inverse of the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

   is given by the formula

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} .$$

5. Find the solution to the linear equation

$$\dot{\boldsymbol{x}} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \boldsymbol{x} , \qquad \boldsymbol{x}(0) = \begin{pmatrix} 2 \\ 5 \end{pmatrix} .$$

6. Suppose that $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$ are solutions of $\dot{\boldsymbol{x}} = A\boldsymbol{x}$. Show that $\boldsymbol{x}(t) = \alpha\, \boldsymbol{x}_1(t) + \beta\, \boldsymbol{x}_2(t)$, where α and β are arbitrary constants, is also a solution.