Introduction to Partial Differential Equations

Homework 4

due March 27, 2017

1. Evans, p. 163 problem 3.

2. (From Evans, p. 164, Question 13.) Let $u \in C(\mathbb{R} \times [0, T])$ for some $T > 0$ be an integral solution to the scalar conservation law

$$
\begin{aligned}
u_t + F(u)_x &= 0 && \text{in } \mathbb{R} \times (0, T), \\
u &= g && \text{on } \mathbb{R} \times \{t = 0\}.
\end{aligned}
$$

   Assume further that for any fixed $t \in [0, T]$, $u(\,\cdot\,, t)$ has compact support in $\mathbb{R}$, and that $F(0) = 0$. Show that

$$
\int_{\mathbb{R}} u(x, t)\, dx = \int_{\mathbb{R}} g(x)\, dx
$$

   for every $t \in [0, T]$.