Operations Research

Homework 7

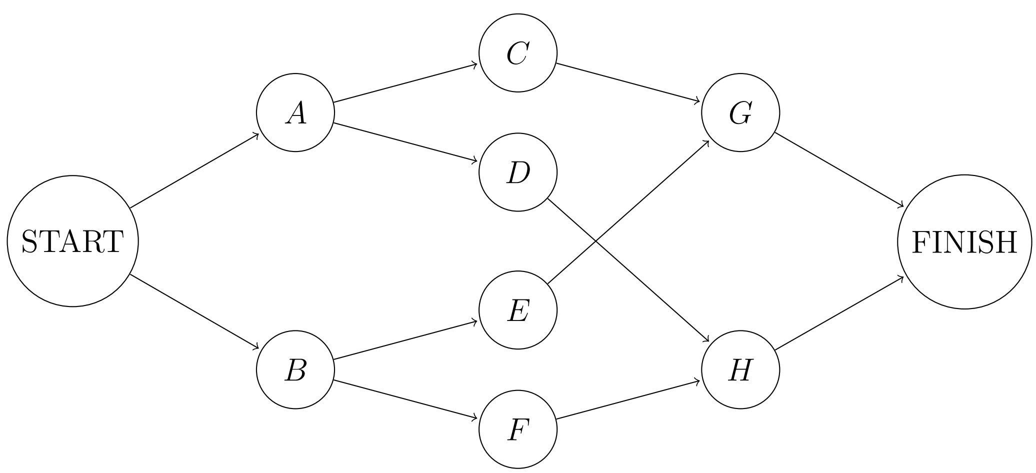
Due in class Wednesday, April 6, 2016

1. Take the Pyomo file `20160318-Reliable-CMPmethod.ipynb` demonstrating the project time crashing optimization problem. Modify the code to

   (a) Compute the minimal time to completion without crashing, and

   (b) Compute the minimal time to completion with full crashing, and the minimal associated extra cost.

2. *(HL, Exercise 9.8-4.)* The 21st Century Studios is about to begin the production of its most important (and most expensive) movie of the year. The movie's producer, Dusty Hoffmer, has decided to use PERT/CPM to help plan and control this key project. He has identified the eight major activities (labeled $A, B, \ldots, H$) required to produce the movie. Their precedence relationships are shown in the project network below.



   Dusty now has learned that another studio also will be coming out with a blockbuster movie during the middle of the upcoming summer, just when his movie was to be released. This would be very unfortunate timing. Therefore, he and the top management of 21st Century Studios have concluded that they must accelerate production of their movie and bring it out at the beginning of the summer (15 weeks from now) to establish it as *the* movie of the year. Although this will require substantially increasing an already huge budget, management feels that this will pay off in much larger box office earning both nationally and internationally.

Dusty now wants to determine the least costly way of meeting the new deadline 15 weeks hence. Using the CPM method of time-cost trade-offs, he has obtained the following data.

Activity	**Normal Time**	**Crash Time**	**Normal Cost**	**Crash Cost**
A	5 weeks	3 weeks	\$24 million	\$36 million
B	3 weeks	2 weeks	\$13 million	\$25 million
C	4 weeks	2 weeks	\$21 million	\$29 million
D	6 weeks	3 weeks	\$30 million	\$50 million
E	5 weeks	4 weeks	\$26 million	\$36 million
F	7 weeks	4 weeks	\$35 million	\$57 million
G	9 weeks	5 weeks	\$30 million	\$53 million
H	8 weeks	6 weeks	\$35 million	\$51 million

Use Pyomo to compute the optimal schedule to achieve completion in 15 weeks.

3. Recall that the maximum flow problem for a network with node set N and arc set A can be formulated as a linear programming problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{(i,T)\in A} x_{iT} \\ \text{subject to} \quad & \sum (i,j) \in A x_{ij} = \sum_{(j,i)\in A)} x_{ji} \text{ for every } j \in N\backslash\{S,T\} \\ & \text{and } 0 \leq x_{ij} \leq u_{ij} \,, \end{aligned} \tag{P1}$$

where S is the source node, T is the final (sink) node, u_{ij} are the maximal capacities of the arcs, and the decision variable x_{ij} is the flow through arc (i,j).

There is an alternative linear programming formulation: let $\mathcal{P}$ denote the set of all paths from S to T. Let x_p denote the flow through the path $p \in \mathcal{P}$. Then the problem can be formulated as follows:

$$\begin{aligned} \text{maximize} \quad & \sum_{p\in\mathcal{P}} x_p \\ \text{subject to} \quad & \sum_{p\colon a\in p} x_p \leq u_a \text{ for every } a \in A \\ & \text{and } x_p \geq 0 \,. \end{aligned} \tag{P2}$$

The sum in the constraint is to be understood as follows: we fix an arc $a \in A$ and then sum over all paths from S to T which contain this selected arc. We also write u_a as shorthand for u_{ij} when $a = (i,j)$.

Formulation (P2) is only of theoretical interest, because the number of paths from S to T typically increases exponentially with the size of the network, so that it does not provide an efficient way to solve the maximum flow problem.

(a) Give an argument why (P1) and (P2) are equivalent.

(b) (P2) has the form of the standard "resource allocation problem". Verify this statement by listing the analogies to *resource*, *activity*, and *profit*.

(c) (P2) has a standard dual. Write it out and interpret the meaning of the dual variables in the network context.