Operations Research

Homework 6

Due in class Friday, March 18, 2016

1. A construction company needs a heavy excavator for a three-year project. Since main-tenance is expensive, it may be cheaper to replace the machine in the interim. The overall net cost (in $1000\,€$) of purchasing the excavator at the beginning of year i and trading it at the end of year j is as follows.

	Trade in end of year		
Purchase start of year	1	2	3
1	25	60	95
2	–	29	65
3	–	–	37

The task is to find the most economical schedule of purchasing and trading in the excavator(s).

(a) Draw a graphical representation and show that this is a shortest path problem.

(b) Solve the problem. This is most easily done by hand in this case.

2. *(From HL, Problem 8.3-2.)* Four cargo ships will be used for shipping goods from one port to four other ports (labeled 1, 2, 3, 4). Any ship can be used for making any one of these four trips. However, because of differences in the ships and cargoes, the total cost of loading, transporting, and unloading the goods for the different ship-port combinations varies considerably, as shown in the following table:

	Port			
	1	2	3	4
Ship 1	$500	$400	$600	$700
Ship 2	$600	$600	$700	$500
Ship 3	$700	$500	$700	$600
Ship 4	$500	$400	$600	$600

The objective is to assign the four ships to four different ports in such a way as to minimize the total cost for all four shipments.

(a) Let $x_{ij} = 1$ if ship i is sent to port j with $x_{ij} = 0$ otherwise. (A problem of this form is called an assignment problem.) Formulate and solve this problem in Pyomo. You should submit a printout of your Ipython notebook showing code and solution.

Hint: In Pyomo, you can declare a variable to take only values 0 and 1 by using the option `within=Boolean`.

(b) Now suppose you drop the requirement that the x_{ij} are Boolean and replace it with the usual non-negativity requirement. In other words, you are allowing to send "a fraction of a ship" to one port and another fraction to another. Re-solve the problem in Pyomo. What do you see?

(c) *Extra credit:* Can you come up with a more general mathematical statement and a proof of what you have seen?

3. Write a Pyomo program to solve the maximum flow problem for the Seervada Park example from Hillier and Lieberman, with the network data reproduced here. The arcs are directed and have a flow capacity as indicated.