Operations Research

Homework 5

Due in class Friday, March 11, 2016

1. The primal form of the abstract *activity analysis problem* is given by

$$\begin{aligned} &\text{maximize } \boldsymbol{c}^T \boldsymbol{x} \\ &\text{subject to } A\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \geq 0\,. \end{aligned} \tag{P}$$

   The corresponding dual problem reads

$$\begin{aligned} &\text{minimize } \boldsymbol{y}^T \boldsymbol{b} \\ &\text{subject to } \boldsymbol{y}^T A \geq \boldsymbol{c}^T, \boldsymbol{y} \geq 0\,. \end{aligned} \tag{D}$$

   Here, A is an $n \times m$ matrix, $\boldsymbol{x}, \boldsymbol{c} \in \mathbb{R}^m$, and $\boldsymbol{y}, \boldsymbol{b} \in \mathbb{R}^n$.

   Throughout, suppose that $\boldsymbol{x}$ is feasible for (P) and $\boldsymbol{y}$ is feasible for (D).

   (a) Show that $(\boldsymbol{y}^T A - \boldsymbol{c}^T)\boldsymbol{x} \geq 0$ and $\boldsymbol{y}^T(\boldsymbol{b} - A\boldsymbol{x}) \geq 0$.

   (b) Suppose further that $\boldsymbol{x}$ is optimal for (P) and $\boldsymbol{y}$ is optimal for (D). Conclude that $(\boldsymbol{y}^T A - \boldsymbol{c}^T)\boldsymbol{x} = 0$ and $\boldsymbol{y}^T(\boldsymbol{b} - A\boldsymbol{x}) = 0$.
   Hint: Use strong duality.

   (c) Vice versa, suppose that $(\boldsymbol{y}^T A - \boldsymbol{c}^T)\boldsymbol{x} = 0$ and $\boldsymbol{y}^T(\boldsymbol{b} - A\boldsymbol{x}) = 0$. Conclude that $\boldsymbol{x}$ is optimal for (P) and $\boldsymbol{y}$ is optimal for (D).
   Hint: Use weak duality, i.e., the result from Homework 4 Question 1(c).

2. *(This is a variation of Exercise 8.1-2 from HL.)* The Childfair Company has three plants producing child push chairs that are to be shipped to four distribution centers. Plants 1, 2, and 3 produce 12, 17, and 11 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance from each plant to the respective distribution centers is given below:

	Distribution Center			
	1	**2**	**3**	**4**
Plant 1	800 miles	1 300 miles	400 miles	700 miles
Plant 2	1 100 miles	1 400 miles	600 miles	1 000 miles
Plant 3	600 miles	1 200 miles	800 miles	900 miles

The freight cost for each shipment is \$100 plus 50 cents per mile. The objective is minimizing the total cost of transportation.

(a) Formulate and solve the problem in Pyomo.

(b) Now suppose that demand in the area served by Center 1 goes up to 15 shipments per month. Production cannot be increased on short notice, so some or all of the distribution centers will be under-supplied. Modify your Pyomo code to determine the total number of shipments to arrive at each of the centers if the objective is still to minimize the overall cost of transportation.

Submit a printout of your Ipython notebook showing code and output for each.

3. *(HL, Exercise 8.1-4.)* The Versatech Corporation has decided to produce three new products. Five branch plants now have excess product capacity. The unit manufacturing cost of the first product would be \$31, \$29, \$32, \$28, and \$29 in Plants 1, 2, 3, 4, and 5, respectively. The unit manufacturing cost of the second product would be \$45, \$41, \$46, \$42, and \$43 in Plants 1, 2, 3, 4, and 5, respectively. The unit manufacturing cost of the third product would be \$38, \$35, and \$40 in Plants 1, 2, and 3, respectively, whereas Plants 4 and 5 do not have the capability for producing this product. Sales forecasts indicate that 600, 1 000, and 800 units of products 1, 2, and 3, respectively, should be produced per day. Plants 1, 2, 3, 4, and 5 have the capacity to produce 400, 600, 400, 600, and 1 000 units daily, respectively, regardless of the product or combination of products involved. Assume that any plant having the capability and capacity to produce them can produce any combination of the products in any quantity. Management wishes to know how to allocate the new products to the plants to minimize total manufacturing cost.

(a) Formulate this problem as a transportation problem.

(b) Use Pyomo to obtain an optimal solution.

Part (a) should be submitted handwritten on paper or typed into the Ipython notebook using mathematical markup; for part (b) submit a printout of your Ipython notebook showing code and output.