

# Operations Research

## Homework 4

Due in class Wednesday, March 2, 2016

1. The primal form of the standard *activity analysis problem* (or *resource allocation problem*) is given by

$$\begin{aligned} & \text{maximize } \mathbf{c}^T \mathbf{x} \\ & \text{subject to } A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0. \end{aligned} \tag{P}$$

The corresponding dual problem reads

$$\begin{aligned} & \text{minimize } \mathbf{y}^T \mathbf{b} \\ & \text{subject to } \mathbf{y}^T A \geq \mathbf{c}^T, \mathbf{y} \geq 0. \end{aligned} \tag{D}$$

Here,  $A$  is an  $n \times m$  matrix,  $\mathbf{x}, \mathbf{c} \in \mathbb{R}^m$ , and  $\mathbf{y}, \mathbf{b} \in \mathbb{R}^n$ .

- (a) Show that if  $\mathbf{x}$  solves (P) and  $\mathbf{y}$  solves (D), then

$$\mathbf{y}^T \mathbf{b} \geq \mathbf{c}^T \mathbf{x}.$$

- (b) Conclude that the primal problem does not have a finite maximum if and only if the feasible region of the dual problem is empty.
- (c) Show that if  $\mathbf{x}$  is feasible for problem (P) and  $\mathbf{y}$  is feasible for problem (D), and if furthermore

$$\mathbf{y}^T \mathbf{b} = \mathbf{c}^T \mathbf{x},$$

then  $\mathbf{x}$  solves (P) and  $\mathbf{y}$  solves (D).

2. (*This is a variation of Exercise 6.8-8 from HL.*) David, LaDeana, and Lydia are the sole partners and workers in a company which produces fine clocks. David and LaDeana each are available to work a maximum of 40 hours per week at the company, while Lydia is available to work a maximum of 20 hours per week. The company makes two different types of clocks: a grand-father clock and a wall clock. To make a clock, David (a mechanical engineer) assembles the inside mechanical parts of the clock while LaDeana (a woodworker) produces the handcarved wood casings. Lydia is responsible for taking orders and shipping the clocks. The amount of time required for each of these tasks is shown below.

<b>Task</b>	<b>Time Required</b>	
	<b>Grandfather Clock</b>	<b>Wall Clock</b>
Assemble clock mechanism	6 hours	4 hours
Carve wood casing	8 hours	4 hours
Shipping	3 hours	3 hours

Each grandfather clock built and shipped yields a profit of \$300 while each wall clock yields a profit of \$200.

- (a) Formulate and solve the problem in Pyomo, and ask Pyomo to compute shadow prices (dual variables) for each activity. Please submit a printout of your Ipython notebook showing code and output.
- (b) Occasionally, someone stops by asking for help with restoring antique clocks. How much should David charge per hour for mechanical repairs and how much should LaDeana charge per hour for wood restoration assuming that they do not wish to add more working hours and also do not wish to reduce company profit if one of them is taking on a repair job?