Introduction to Partial Differential Equations

Homework 5

due March 16, 2015

1. Evans, p. 87 problem 12.

2. Evans, p. 87 problem 13.

3. Let $u(x,t)$ solve the heat equation

$$\begin{aligned} u_t - \Delta u = 0 \qquad & \text{in } \mathbb{R}^n \times (0,\infty)\,, \\ u = g \qquad & \text{on } \mathbb{R}^n \times \{t=0\} \end{aligned}$$

with $g \in C(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$. Show that

$$\|u\|_{L^\infty} \to 0 \qquad \text{as } t \to \infty$$

while

$$\int_{\mathbb{R}^n} u(x,t)\, dx = \text{const}\,.$$

Give a physical interpretation of each of the statements.

4. Find a solution formula for the heat equation with advection,

$$\begin{aligned} u_t - \Delta u + b \cdot Du = 0 \qquad & \text{in } \mathbb{R}^n \times (0,\infty)\,, \\ u = g \qquad & \text{on } \mathbb{R}^n \times \{t=0\}\,. \end{aligned}$$

Hint: which equation is solved by $v(x,t) = u(x+bt,t)$?