Introduction to Partial Differential Equations

Homework 3

due February 25, 2015

1. Evans, p. 86 problem 4.
2. Evans, p. 86 problem 5.
3. In class we have used Liouville's theorem to show that any bounded solution of the Poisson equation
$$-\Delta u = f$$
for $f \in C^2_{\mathrm{c}}(\mathbb{R}^n)$, $n \geq 3$, is given by the solution formula
$$u(x) = \int_{\mathbb{R}^n} \Phi(x-y)\, f(y)\, dy$$
up to an arbitrary additive constant. (Evans, p. 30, Theorem 8.)

   This statement does not hold in dimension $n = 2$ since solutions are generically unbounded. Use Liouville's theorem to conjecture and prove the corresponding theorem for $n = 2$.
4. Let
$$K(x,y) = \frac{2}{n\alpha(n)} \frac{x_n}{|x-y|^n}$$
be the Poisson kernel for the half-plane. Show that
$$\int_{\partial\mathbb{R}^n_+} K(x,y)\, dy = 1$$
for any $x \in \mathbb{R}^n_+$.