Introduction to Partial Differential Equations

Homework 2

due February 18, 2015

1. Show that
$$\Delta u = 2n \lim_{\varepsilon \to 0} \frac{1}{\varepsilon^2} \fint_{\partial B(x,\varepsilon)} (u(y) - u(x)) \, dS(y) \,.$$

2. Evans, p. 85 problem 3.

   Note: This problem can be hard when you try it. Please ask in class in case you get stuck.

3. (a) The *standard mollifier* is defined by
$$\eta(x) \equiv \begin{cases} c(n) \, \exp\left(\dfrac{1}{|x|^2 - 1} \right) & \text{if } |x| < 1 \\ 0 & \text{otherwise} \,, \end{cases}$$
   where $c(n)$ is chosen such that
$$\int_{\mathbb{R}^n} \eta(x) \, dx = 1 \,.$$
   Show that $\eta \in C^\infty(\mathbb{R}^n)$.

   (b) Show that if η_ε is a radial mollifier, and u is a radial, locally integrable function, then its mollification
$$u_\varepsilon(x) = (\eta_\varepsilon * u)(x) = \int_{\mathbb{R}^n} \eta_\varepsilon(y) \, u(x-y) \, dy$$
   is also radial.

4. In the proof of Evans, p. 23, Theorem 1, we have used that
$$D \int_{\mathbb{R}^n} \Phi(y) \, f(x-y) \, dy = \int_{\mathbb{R}^n} \Phi(y) \, Df(x-y) \, dy \,.$$
   State sufficient assumptions under which this manipulation is permitted, and why.