General Mathematics and CPS II

Exercise 21

April 29, 2015

There is no class Friday, May 1. This homework set is due Wednesday, Mai 6 in class.

1. Let $u\colon [0, 2\pi] \to \mathbb{R}$ be defined via the Fourier series

$$u(x) = \sum_{k\in\mathbb{Z}} \hat{u}_k \, \mathrm{e}^{\mathrm{i}kx} \, .$$

   Show that its derivative $w(x) = u'(x)$, assuming it exists, has Fourier coefficients

$$\hat{w}_k = \mathrm{i}k \, \hat{u}_k \, .$$

2. Handout, Exercise 3.

3. Suppose a smooth periodic function $u\colon [0, 2\pi) \to \mathbb{C}$ is sampled at the N equidistant points $x_j = jh$ with $h = 2\pi/N$ and $j = 0, \ldots, N-1$. We set $u_j = u(x_j)$, take the discrete Fourier transform

$$\tilde{u}_k = \sum_{j=0}^{N-1} \mathrm{e}^{-\mathrm{i}kx_j} \, u_j \, ,$$

   and define the reconstruction of u via

$$v(x) = \sum_{k=-N/2}^{N/2-1} \mathrm{e}^{\mathrm{i}kx} \, \tilde{u}_k \, .$$

   Show that

$$\hat{v}_k = \begin{cases} \tilde{u}_k & \text{for } k \in -N/2, \ldots, N/2-1 \, , \\ 0 & \text{otherwise} \, . \end{cases}$$