General Mathematics and CPS II

Exercise 16

April 10, 2015

1. Find all solutions for the underdetermined linear system $A\boldsymbol{x} = \boldsymbol{b}$, where

$$A = \begin{pmatrix} 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 3 & 3 & 2 & 1 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{b} = \begin{pmatrix} -1 \\ 1 \\ -2 \\ 0 \end{pmatrix}.$$

2. Consider a linear programming problem with n variables in standard form. Explain why a non-negative solution to $m \leq n$ equality constraints in which at least $n - m$ variables are zero represents a vertex of the feasible region.

   Hint: Recall how the general solution to an underdetermined system of linear equations looks like, cf. Question 1. You may begin by visualizing the feasible region in two and three dimensions first.

   Remark: These $n - m$ nonzero variables are the *basic variables* in the description of the simplex method.

3. Write the following linear programming problem in its standard form.

   Maximize
   $$z = 2\,x_1 - x_2 + x_3$$

   subject to

   $$x_1 - x_2 \leq 1\,,$$
   $$x_2 - x_3 \geq 1\,,$$
   $$x_3 - x_1 \leq 3\,,$$
   $$x_1 \geq -2\,,$$
   $$x_2 \leq 1\,,$$
   $$x_3 \leq 0\,.$$