

General Mathematics and CPS II

Exercise 14

March 27, 2015

1. Kac ring paper, Exercise 14.
2. Fix $N \in \mathbb{N}$. Let S denote the $N \times N$ *circular shift matrix*

$$S = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}$$

Further, let M denote the diagonal matrix

$$M = \begin{pmatrix} m_1 & 0 & & & \\ 0 & m_2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & 0 \\ & & & 0 & m_N \end{pmatrix}$$

where $|m_1| = \cdots = |m_N| = 1$.

Show that $\{(SM)^i : i \in \mathbb{Z}\}$ is a group. What is its order?

3. There is a correspondence of the construction in Questions 3 with the Kac ring model. Explain! Further, there is a correspondence of this construction with glide reflections. Explain!