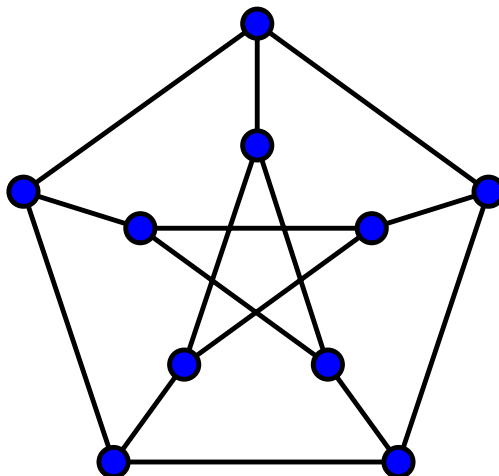


General Mathematics and CPS II

Exercise 3

February 11, 2015

1. Let G be a finite connected planar graph with V its set of vertices, E its set of edges, and F its set of faces.
 - (a) Show that $2|E| \geq 3|F|$.
 - (b) Show that $|E| \leq 3|V| - 6$.
 - (c) Conclude that every planar graph must have at least one vertex of valency less than 6.
2. The *Petersen graph* can be constructed as the graph whose vertices are two-element subsets of a five-element set, and two vertices are joined by an edge whenever the corresponding two-element sets are disjoint. It can be visualized as follows.¹



Show that the Petersen graph cannot be embedded in the plane.

¹Image from https://commons.wikimedia.org/wiki/File:Petersen1_tiny.svg