Partial Differential Equations

Homework 2

due March 4, 2014

1. Let U be open. Find an exponent $\theta \in (0,1)$ such that

   $$\|u\|_{L^q} \leq \|u\|_{L^p}^{\theta} \, \|u\|_{L^r}^{1-\theta}$$

   for all $u \in L^p(U) \cap L^r(U)$ and $1 \leq p \leq q \leq r \leq \infty$.

2. Evans, p. 290, Problem 6

3. Evans, p. 290, Problem 8

4. Evans, p. 290, Problem 9

5. Show that the Rellich–Kondrachov is sharp, i.e., that $W^{1,p}$ is not compactly embedded into L^{p^*} where $p^* = np/(n-p)$ is the Sobolev conjugate.

   Hint: Consider sequences of dilations

   $$u_m(x) = m^{\alpha} \, u(mx)$$

   of a compactly supported function $u \in W^{1,p}$.

6. Evans, p. 291, Problem 10

   Hint: Look at the proof of Morrey's inequality in Evans (not covered in class).