General Mathematics and CPS II

Exercise 6

February 21, 2014

1. (Ivanov, p. 35, Exercise.) Prove that the composite of two axial symmetries (an *axial symmetry* is a reflection about a line, the *axis*) with intersecting axes of symmetry is a rotation about the point of intersection of these axes through an angle equal to twice the angle between them.

2. (Ivanov, p. 34, Problem 9.) Consider two squares situated in the same plane. Join any corner of one square to any corner of the other by means of a line segment, and then, proceeding in the same direction around both squares, the next corner to the next, and so on.

   Prove that the midpoints of the four line segments so constructed are also the vertices of a square.

3. A *Euclidean transformation* F or, in short, a *motion* of the plane is a map which preserves the Euclidean distance between points, i.e.

$$|F(a) - F(b)| = |a - b|$$

   where $|a - b|$ denotes the distance between two arbitrary points a and b.

   Show that every motion of the plane is bijective.

   Hint: To prove that it is surjective, notice that the transformation must map lines onto lines and circles onto circles.