# Engineering and Science Mathematics 2B

Homework 6

due April 3, 2013, before 12:00

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. Let $f$ be a periodic function on the interval $[-\pi, \pi]$ with Fourier representation

$$f(x) = \sum_{k=-\infty}^{\infty} c_k \, \mathrm{e}^{\mathrm{i}kx} \qquad \text{where} \qquad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-\mathrm{i}kx} \, f(x) \, \mathrm{d}x \, .$$

   Show that when $f$ is real, then $c_k^* = c_{-k}$.

2. Let $f$ be a periodic function on the interval $[-\pi, \pi]$ with Fourier representation

$$f(x) = \sum_{k=-\infty}^{\infty} c_k \, \mathrm{e}^{\mathrm{i}kx} \qquad \text{where} \qquad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-\mathrm{i}kx} \, f(x) \, \mathrm{d}x \, .$$

   Express the Fourier coefficients of

   (a) $f(x - x_0)$ where $x_0$ is a constant,

   (b) $f(-x)$,

   (c) $f^*(x)$

   in terms of the Fourier coefficients $c_k$ of $f$.

3. Let $f$ be a periodic function on the interval $[-\pi, \pi]$ with Fourier representation

$$f(x) = \sum_{k=-\infty}^{\infty} c_k \, \mathrm{e}^{\mathrm{i}kx} \qquad \text{where} \qquad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-\mathrm{i}kx} \, f(x) \, \mathrm{d}x \, .$$

   Express the Fourier coefficients of

   (a) $f'(x)$,

   (b) $\int_{-\pi}^{x} f(\xi) \, \mathrm{d}\xi$ assuming that $c_0 = 0$

in terms of the Fourier coefficients $c_k$ of $f$.

4. Compute the complex Fourier series of the function $f(x) = \mathrm{e}^x$ on the interval $[-\pi, \pi]$.

5. (E) Compute the complex Fourier series of the function $f(x) = |x|$ on the interval $[-\pi, \pi]$.

   (A) In addition to (E), express $f$ as a Fourier cosine series.

6. (E) Let

$$\boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix} .$$

   Compute the projection of $\boldsymbol{v}$ onto the subspace spanned by the orthonormal vectors

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} , \quad \boldsymbol{e}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} .$$

   (A) An odd function of period $2\pi$ is approximated by a Fourier sine series having only $N$ terms. The error in the approximation is measured by the mean-square deviation

$$E_N = \int_{-\pi}^{\pi} \left[ f(x) - \sum_{n=1}^{N} b_n \sin nx \right]^2 \mathrm{d}x .$$

   By differentiating $E_N$ with respect to the coefficients $b_n$, find the values of $b_n$ that minimize $E_N$.