Engineering and Science Mathematics 2B

Review for Final Exam

Thursday, May 23, 12:30-14:30 in the Campus Center East Wing and Conference Hall

1. Solve a system of linear equations, matrix inversion.

   Study problems: There are plenty of practice problems on homework, past exams, and in the book.

2. Concept of vector space, linear independence, basis; linear transformations: definition, representation by a matrix; change of basis.

   Study problems: See, in particular, Question 6 on Midterm 1 from Spring 2003, Question 6 on Midterm 1 from Spring 2004, and Homework 4 Question 2.

3. Inner products: Definition, examples, orthonormal basis, projections.

4. Gram-Schmidt orthonormalization.

   Study problem: Midterm 2 Retake, Question 2.

5. Can you show that the eigenvectors of a symmetric (Hermitian) matrix are orthogonal provided the eigenvalues are distinct?

6. Eigenvalues and eigenvectors, in particular for Hermitian matrices; determinants; diagonalization.

   Study problem: Midterm 1 Question 5A (some versions of the exam have a different order of questions – this reference is uses the order in the file `midterm1-solutions.pdf`).

7. Fourier Series: compute the series, properties. Concentrate on the complex Fourier series!

   Study problems: There are plenty of practice problems on homework, past exams, and in the book.

8. Fourier transform: Compute the transform, inverse Fourier transform, properties, Fourier transform of the convolution of two functions, Parseval theorem.

9. Delta function: basic properties and simple computations.

10. Probability: Outcomes, events, sample spaces, definition of probability, conditional probability.

11. Bayes' rule.

    Study problems: Think through the Monty Hall problem and its formalization via Bayes' rule again; Homework 8 Question 6, and the following problem (which is slightly larger than the problems which will be on the exam, but gives a rather typical Bayes' rule setting):

    A machine sorts potatoes into small, medium and large ones, but it makes mistakes. Suppose that the probability that a random potato will end up in a specified category is according to the following table.

	categorized as		
potato of size	small	medium	large
small	0.8	0.15	0.05
medium	0.2	0.7	0.1
large	0.05	0.1	0.85

    Suppose further that 30 % of the potatoes are small, 45 % are medium and 25 % are large. What are the percentages of small, medium and large potatoes in each of the three piles the machine produces?

12. Permutations and Combinations; their use for the computation of probabilities; expected value and variance; binomial, Poisson, and Gaussian distribution; know how to compute the mean and variance using the moment generating function.

    Study problem: In a game you throw a pair of dice. If the sum of the values equals 12 you win 5 euros, if it equals 11 you win 1 euro; for all other outcomes you won't get anything.

    (a) Calculate the probability of winning exactly 6 euros after 6 games.

    (b) Compute the expected value of the pay-out per game.

13. Properties of random variables, probability distribution functions for continuous random variables, sums of identically and independently distributed random variables ("the road to the central limit theorem").

    Some questions to ask: If X and Y are random variables, determine if, or under what conditions, the following statements are true: $E[XY] = E[X]\,E[Y]$, $\mathrm{Var}[XY] = \mathrm{Var}[X]\ \mathrm{Var}[Y]$, $E[X+Y] = E[X] + E[Y]$, $\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$?