Nonlinear Dynamics Lab

March 18–19, 2013

Report due April 2, 2013

This weeks lab sessions study *Chua's Circuit*, a nonlinear electric oscillator which can be assembled with standard electronics parts. For background and circuit diagrams, consult the attached article by Hobson and Lansbury.

In practice, inductors have a non-negligible resistance which we denote R_L which has significant ramifications for the functioning of the circuit. It can be analyzed as a resistor in series with an ideal inductor.

Consequently, the differential equations describing Chua's Circuit read

$$C_1 \frac{\mathrm{d}V_1}{\mathrm{d}t} = \frac{V_2 - V_1}{R_\mathrm{c}} - I_\mathrm{nl}\,, \tag{1}$$

$$C_2 \frac{\mathrm{d}V_2}{\mathrm{d}t} = \frac{V_1 - V_2}{R_\mathrm{c}} + I_L\,, \tag{2}$$

$$L \frac{\mathrm{d}I_L}{\mathrm{d}t} = -V_2 - R_L\, I_L\,. \tag{3}$$

The resistance R_c is the coupling resistor, labeled $1/G$ in Hobson and Lansbury's article. A short derivation of these equations should be contained in your lab report.

Lab tasks

1. The inductor we have readily available is one with 10 mH. Take a multimeter and measure its resistance R_L.

2. Assemble Chua's circuit. Since the inductor is different from the one used in *Hobson and Lansbury*, some resistances should be chosen differently from the circuit diagram: take $R_1 = 2.2\,\mathrm{k\Omega}$ and $R_4 = 1\,\mathrm{k\Omega}$, the other values as in the diagram. For the variable resistance R_c we use a multiturn potentiometer which allows fine control of its resistance.

3. Determine experimentally the value of R_c which corresponds to the onset of chaos. Do you see a sharp transition, or a period doubling cascade as for the logistic map?

4. Measure the response curve of each of the nonlinear inverse resistors. To do so, use a third operational amplifier as a voltage follower attached to the noninverting inputs

of the first two operational amplifiers. To the output of this voltage follower, you can attach the oscilloscope ground. The X- and Y-channels of the oscilloscope can then be attached to ground to measure $-V_1$ and to the far end of R_3 (or R_6). The voltage across R_3 resp. R_6 can be converted into the current response of the corresponding nonlinear resistor via Ohm's law.

5. If you have time: Use an inductor with 100 mH, also available in the lab, and try if you can—potentially modifying the values of the capacitors or resistors as well—find a regime as well. If this is successful, you will likely obtain less hysteresis in the response curve of the nonlinear resistors, and consequently better agreement of theory and experiment. (Note: the 100 mH inductor might have too big a resistance to excite nonlinear oscillations, so this is not guaranteed to work!)

Report items

1. Analyze the "inverse resistor" consisting of R_1, $R_2 = R_3$, and the operational amplifier. Write out an expression for the current response as a function of the input voltage, assuming that the operational amplifier saturates at output voltage $\pm V_{\text{max}}$. Write out and plot an expression for the overall response curve of the two parallel inverse resistors used in Chua's circuit.

2. Use the two experimentally response curves to obtain an approximate measured response curve. Plot the theoretical curve and the measured curve in one coordinate system.

3. Write a program which simulates Chua's Circuit over a time interval $T = 0.02$. Plot V_1 vs. V_2 for times $t = [T/2, T]$, thereby discarding transients. Find the value for R_{c} at the onset of chaos. (Both for the theoretical response curve and for the reconstruction of the measured curve where you may ignore hysteresis effects.)

4. Compare the experimental with the numerical results and discuss possible differences.

The experiment and the lab report may be done in groups of two.