Nonlinear Dynamics Lab

Session 7 and 8

due March 4, 2013

1. Consider the equation for the mathematical pendulum

$$\dot{q} = p$$
$$\dot{p} = -\sin q\,.$$

   Write a program that solves the mathematical pendulum with

   (a) the explicit Euler method,
   (b) the implicit Euler method,
   (c) the implicit midpoint method.

   Take initial values $q(0) = 0$ and $p(0) = 1.9$ and plot example solutions $q(t)$ vs. t for each method into one graph for a time horizon of many oscillation periods.

2. Use your program from Exercise 1 to plot the solution trajectories in the q-p phase plane and overlay it with a contour plot of the energy

$$E = \frac{1}{2}\,p^2 - \cos q\,.$$

3. Read up on the build-in ODE solvers in `scipy.integrate.ode`. Obtain a reference solution for the mathematical pendulum using one of these integrators for a fixed final time, e.g. $T = 10$, and determine the order of the previously implemented methods by plotting the error (the difference of the result of your solver vs. that of the built-in reference solver) vs. the step size on a doubly logarithmic scale.

4. Solve the van der Pol equation

$$\dot{x} = y\,,$$
$$\dot{y} = \mu\,(1 - x^2)\,y - x$$

   for $\mu = 1000$ with different built-in solvers. Be sure to involve at least one explicit solver and one implicit one.

You should submit by the deadline the runable code for each problem as well as a written description of your observations as well as a discussion of these observations for Exercises 1/2 (together) and 4.