General Mathematics and CPS II

Exercise 21

May 9, 2012

1. Let G be a finite abelian group of order N, written additively, and let $\hat{G}$ denote its dual group. Show that for every nonzero $a \in G$,

$$\sum_{\chi \in \hat{G}} \chi(a) = 0\,.$$

Then conclude that

$$\sum_{\chi \in \hat{G}} \overline{\chi(a)}\chi(b) = \begin{cases} N & \text{if } a = b\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Hint: Mimic the proofs of the corresponding statements on G as given in class.

2. Let $f_A \in \mathbb{C}^G$ denote the characteristic function of a set $A \subset G$, i.e.,

$$f_A(a) = \begin{cases} 1 & \text{if } a \in A\,, \\ 0 & \text{if } a \notin A\,, \end{cases}$$

and $\hat{f}_A \in \mathbb{C}^{\hat{G}}$ its Fourier transform. Set

$$\Phi(A) = \max\{|\hat{f}_A(\chi)|\colon \chi \in \hat{G}, \chi \neq \chi_0\}\,.$$

(Recall that χ_0 denotes the principal character where $\chi_0(a) = 1$ for all $a \in G$.)

Show that $\Phi(A) = \Phi(G \setminus A)$.